Research Article https://doi.org/10.12973/eu-jer.11.1.1

European Journal of Educational Research
Volume 11, Issue 1, 1 - 16.
ISSN: 2165-8714
https://www.eu-jer.com/

Developing Primary Students' Understanding of Mathematics through Mathematization: A Case of Teaching the Multiplication of Two Natural Numbers

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Received: August 16, 2021 ▪ Revised: September 25, 2021 ▪ Accepted: October 12, 2021

Abstract: Numeracy is one of the essential competencies that the objectives of teaching math to primary students should be towards. However, many research findings show that the problem of "innumeracy" frequently exists at primary schools. That means children still do not feel at home in the world of numbers and operations. Therefore, the paper aims to apply the realistic mathematics education (RME) approach to tackling the problem of innumeracy, in the case of teaching the multiplication of two natural numbers to primary students. We conducted a pedagogical experiment with 46 grade 2 students who have not studied the multiplication yet. The pedagogical experiment lasted in six lessons, included seven activities and nine worksheets which are designed according to fundamental principles of RME by researchers. This is mainly a qualitative study. Based on data obtained from classroom observations and students' response on worksheets, under the perspective of RME, the article pointed out how mathematization processes took place throughout students' activities, their attitudes towards math learning, and their learning outcomes. The study results found that students were more interested in math learning and understood the concepts of multiplication of two natural numbers.

Keywords: *Innumeracy, mathematization, multiplication of two natural numbers, realistic mathematics education.*

To cite this article: Hong Duyen, N. T., & Loc, N. P. (2022). Developing primary students' understanding of mathematics through mathematization: A case of teaching the multiplication of two natural numbers. *European Journal of Educational Research*, *11*(1), 1-16. https://doi.org/10.12973/eu-jer.11.1.1

Introduction

Numeracy is the ability to comprehend and manipulate numerical data in everyday life. In other words, it is the ability to confidently use and apply basic mathematics in real-life situations and the workplace (Askew et al., 1997; Cockcroft, 1982; Treffers, 1991a). Competence in numeracy is one of the important aspects assessed by international student assessment programs as PISA (Program for International Student Assessment), IMAS (International Mathematics Assessment for Schools). Thus, the objectives of teaching math to primary students should also be towards the goals of numeracy (Chau et al., 2017).

According to Van den Heuvel-Panhuizen (2008), students need to feel at home in the world of numbers and operations. Numeracy, therefore, is both calculation and mathematics. However, many research findings show that children can successfully perform mathematical procedures without possessing any things like "understanding" (Baroody & Ginsburg, 1986; Sophian, 1995). And, children's understanding is not always manifest in their problem-solving activities (Gelman & Gallistel, 1978; Siegler & Crowley, 1994). These manifestations in children lead teachers to consider children have not understood mathematics yet. Thus, they have not achieved the goal of numeracy. This problem frequently exists at primary schools (Treffers, 1991a). Until now, there have been some research results relating to this problem. For example, some primary students could accurately find the product of 245 × 37 by the long multiplication method, but they could not explain how they did (Armanto, 2002). In addition, first graders at primary schools could not make arguments about how to use addition algorithms, and they did not understand the relationships between numbers (Putra et al., 2011). Besides, students at some primary schools mainly followed the model or imitated when learning four operations with natural numbers, and they did not understand the meaning of the calculation. They could read the whole multiplication table in sequence, but they could not find the result of any calculation in the table without reciting the table (Oanh, 2016).

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In addition, the research team conducted a survey (see Figure 1) in 2019 with the participation of 155 grade 4 students. Obtained data showed that there were 126 students (accounting for 82.3%) correct in question 1, meanwhile only 13 students (8.3%) were correct in question 2. The survey found that grade 4 students became dependent on the long multiplication method, hence, they found difficult to try another strategy when they were constrained by not using that method.

Bài toán 1: Đặt tính rồi tính: 56×47

Bài toán 2: Không đặt tính, em có thể tìm được kết quả của 56×23

? Nếu tìm được, hãy giải thích cách làm của em.

Figure 1. Questions in the survey

(The translation of the Figure 1: Question 1: Use the long multiplication method to find the result of 56×47. Question 2: Not use the long multiplication method, can you find the result of 56×23? If possible, let explain your solution).

Hence, the problem pointed out by Treffers still exists. The main reason is no room for context because algorithms are high on teacher's priority (Treffers, 1991a). Treffers (1991a) also suggested that teachers need to take a realistic approach to teach math. That means teachers teach math according to the theory of Realistic mathematics education (RME).

The philosophy of RME is that students should develop their understanding of mathematics by working with contexts that make sense to them (Dickinson & Hough, 2012). That means necessary conditions for teaching math according to RME are contexts implying mathematics knowledge. Contextual problems are defined as "problems where the problem situation is experientially real to the student" (Gravemeijer & Doorman, 1999, p.111). These contexts help students to imagine and visualize abstract concepts. It is essential for students, especially primary students. The responsibility for providing such contexts belongs to teachers. Thus, the teacher needs to find a suitable series of contextual problems that can map out a possible learning path and in which students can be "guided reinvent" through "progressive mathematization" as a gradual change (Gravemeijer, 1994, 1999). In other words, the teacher needs to map out a conjectured learning trajectory (Gravemeijer, 1994). A conjectured learning trajectory is also called as learning design. It includes learning activities as well as objectives of these activities and the teacher's conjecture how students learn and think. It is also called as Hypothetical Learning Trajectory (HLT) by Simon (1995).

"Guided reinvention" and "mathematization" are two basic terminologies of RME. "Guided reinvention" means that students, under the teacher's guidance if necessary, learn math by doing to discover something new - mathematics (just unknown for themselves, well-known for teacher). The process in which students take action to reinvent mathematics is called "mathematization". In addition, Treffers (1987) classified it into two types: horizontal mathematization and vertical mathematization. A horizontal mathematization is a process in which students use their solutions to describe a contextual problem by symbols. These symbols can be mathematics (formulas, algorithms, etc.) or not (figures, diagrams, etc.). Students can explicitly present them, or even they can visualize or imagine. In the world of symbols, students continue to use their known mathematics to take action to find out new mathematics knowledge. It helps students answer to the problem situated in the context problem. The process in which students take action within the mathematics system is called vertical mathematization (Loc & Duyen, 2017; Loc & Hao, 2016; Loc & Tien, 2020; Menon, 2013; Van den Heuvel-Panhuizen, 2000, 2002). Freudenthal (1991) argued that the distinction between these two processes is rather vague, because the frontier between the world of life and that of symbols is rather vaguely marked.

Besides, the theory of RME also shows the principle of self-developed models. According to this principle, the teacher should bring opportunities to students to use and develop their models (called model-of) in solving specific contextual problems, then to generalize and formalize them into a general model (called model-for) to approach formal knowledge. In other words, this principle plays a significant role in bridging the gap between informal and formal knowledge (Gravemeijer, 1994, 1999). It can be said that RME is a potential approach to attain the goal of numeracy. Indeed, math learning under the perspective of RME will bring students two benefits: Knowledge and ability, when acquired by one's activities, stick better and are more readily available than when imposed by others; math learning may be motivating and exciting (Freudenthal, 1991). According to Treffers (1991b), it is possible for students to reinvent knowledge under the teacher's guidance, because math knowledge can be developed from learners' informal knowledge. Moreover, the perspective of RME is also consistent with the orientation of teaching math towards developing learners' competencies (Anh & Cuong, 2020).

Methodology

Research Questions

To develop primary students' understanding of mathematics, we carried out our study to apply the RME approach, in a case of teaching the multiplication of two natural numbers. In particular, we designed a HLT for learning the multiplication of two natural numbers, underpinned by RME approach. The paper will answer the following two questions:

(1) Does the designed HLT generate an environment in which students perform mathematization to gain mathematics knowledge?

(2) How does the designed HLT affect students' understanding of the multiplication of two natural numbers?

Researchers performed the study in three following stages.

Before conducting a teaching experiment

In this stage, the authors designed the HLT according to a process similar to the first phase of Slow Design mentioned by de Lange (2015), as following steps:

First, researchers performed an epistemological analysis of the multiplication of two natural numbers. It is a necessary step before designing a process of teaching the multiplication of two natural numbers. An epistemological analysis of the knowledge is the fundamental methodological issue relating to creating situations for teaching that knowledge (Chau, 2017). Next, following the fundamental principles of RME and the result of the epistemological analysis, authors mapped out self-developed models in teaching multiplication of two natural numbers to grade 2 students, as shown in Figure 2.

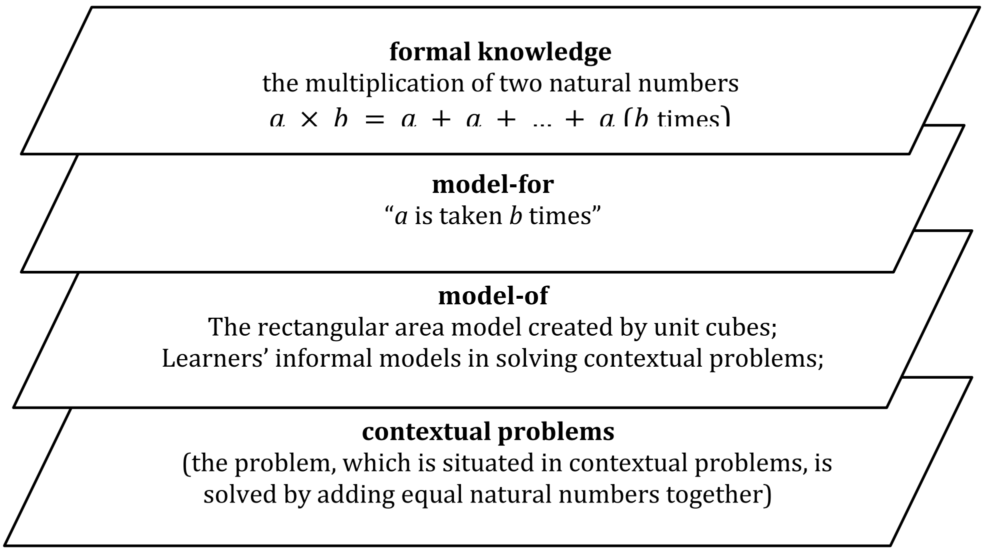


Figure 2. Self-developed models in teaching multiplication to grade 2 students

Based on self-developed models, researchers continued to design a HLT for learning the multiplication of two natural numbers to primary students. The HLT includes seven activities as well as their objectives, as shown in Table 1.

In the current Mathematics Education Curriculum in Vietnam, the multiplication of two natural numbers since Grade 2 (Vietnam Ministry of Education and Training, 2018). Thus, the class 2A was chosen to conduct a pedagogical experiment with participation of 46 students. These students have not studied the multiplication of two natural numbers yet. They were chosen because they and their teacher were willing to participate in the teaching-learning process.

Table 1. The HLT for multiplication of two natural numbers to grade 2 students

No.	Activity	Objectives
1	Students solve contextual problems.	Through mathematization, students will realize: - The relationship between the action "a is taken b times" and the operation ""$a \times b = a + a + \dots + a$ (b times)" - Knowledge the teacher wants to teach: in contextual problems "there are b groups of object X and each group has a objects X", in order to find the total number of objects X, we will plus numbers a together (b times).
2	The teacher introduces the concepts of the multiplication of two natural numbers, the sign "×".	Students know $a \times b$ is a new operation. It is written shortly from $a + a + \dots + a$ (b times). That means "a is taken b times".
3	The teacher introduces the rectangular area model formed by math-link cubes (each rectangular area pattern corresponds to a multiplication of two natural numbers).	Through self-developed models, students can experience successfully guided reinvention to gain the mathematics knowledge "$a \times b = a + a + \dots + a$ (b times)"
4	Students write the corresponding multiplications to the rectangular area models.	
5	Students show the corresponding rectangular area pattern to a given multiplication.	
6	Count by adding n ($n = 2, 3, 4, 5$)	Continue to reinforce for students to understand the concept of multiplication of two natural numbers and apply them to solving contextual problems.
7	Solve problems related to the meaning of multiplication, skills to practice multiplication in practical situations	

Seven activities in the designed HLT were outlined how to organize in class (showed in lesson plans) and the HLT were adjusted after discussing with one experienced teacher, who has more than 13 years of teaching experience and has a Master degree of Mathematics.

Along to the lesson plans, nine worksheets were designed to help students improve their understanding of the multiplication of two natural numbers. In addition, they were also used as tools for collecting students' responses for evaluation purposes. The left column in Table 2 shows criteria for students in grade 2 when they learn the multiplication of two natural numbers. Criteria are requirements to be met, which are taken from the current Math curriculum (Vietnam Ministry of Education and Training, 2018).

Table 2. Criteria and tools

No.	Criteria	Tools for collecting data
1.	Know the meaning of multiplication	Worksheets: 1, 2, 3 and 6
2.	Identify the components of multiplication	Worksheet 5
3.	Recognize the practical meaning of multiplication through pictures, drawings, or real-life situations	Worksheets: 3, 4 and 7
4.	Solve a number of problems associated with solving one-step problems involving the practical meaning of the multiplication of two natural numbers.	Worksheets: 8 and 9.

Then, researchers discussed the whole HLT, the particular lesson plans and nine worksheets with the teacher of the class 2A. The teacher has more than 10 years of teaching experience.

While conducting a teaching experiment

Corresponding to the designed HLT, there were six lessons in the teaching experiment, which took place from January 12, 2021, to January 22, 2021 and were captured on videos while classroom observations were performing, as shown in Table 3.

Table 3. Six pedagogical experimental lessons

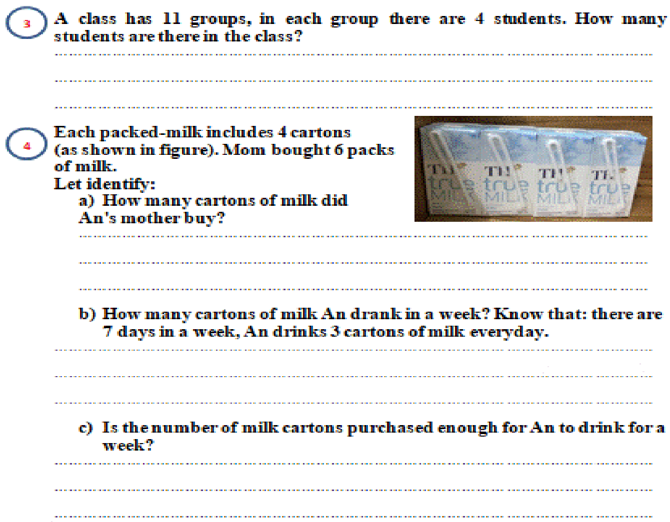
Lesson Title	Multiplication	Factor-Product	The 2 Times Table	The 3 Times Table	The 4 Times Table	The 5 Times Table
Date	January 12	January 13	January 14	January 18	January 20	January 22
Code	M	F-P	2TT	3TT	4TT	5TT
Activities in HLT	1, 2	3, 4,5	4, 5, 6, 7	4, 5, 6, 7	4, 5, 6, 7	4, 5, 6, 7
Worksheets	1, 2, 3, 4	5, 6	7	8	9	

After conducting a pedagogical experiment

Based on these videos, researchers continued to record the observed teaching-learning process. In addition, students' responses on nine worksheets also were collected. Based on data obtained from classroom observations and worksheets, under the perspective of RME, the article aims to analyze qualitatively to point out how mathematization processes took place throughout students' activities, their attitudes towards math, and their learning outcomes.

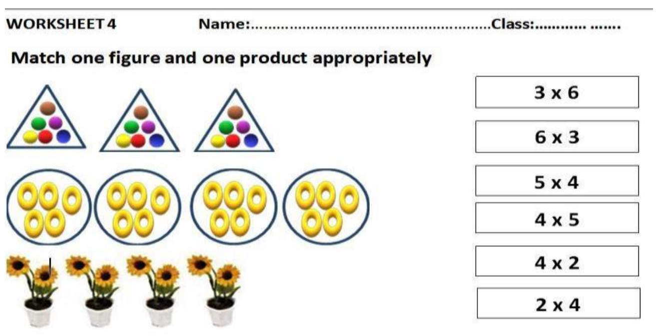
Evaluating the effectiveness of the teaching process (corresponding to the designed HLT)

Researchers decided to use a one-group pre- and post- tests design in evaluation the effectiveness of the teaching process. This method is often used by many educational researchers and was discussed on its methodology by Marsden and Torgerson (2012). In this study, the fourth worksheet was used at the end of the first lesson as the pre- test (Figure 3). And three questions in the ninth worksheet were used at the end of the teaching process as the post- test (Figure 4). The implementing teaching process was considered as an intervention. The validity of the pre- and post- tests is covered by their academic contents and cognitive level they test. According Bloom's Taxonomy, in comparison with Table 2, the pre-test evaluates the students' ability at the second category (Comprehension), meanwhile the post-test evaluates at a higher level, the third one (Application) (Bloom et al., 1956). Obviously, if one student fails the pre-test, but he passes the post-test, there will be a significant improvement in his understanding. Evaluating the validity of the test by its content is also suggested by Thao et al. (2020).


3 A class has 11 groups, in each group there are 4 students. How many students are there in the class?

4 Each packed-milk includes 4 cartons (as shown in figure). Mom bought 6 packs of milk.
Let identify:
a) How many cartons of milk did An's mother buy?
b) How many cartons of milk An drank in a week? Know that: there are 7 days in a week, An drinks 3 cartons of milk everyday.
c) Is the number of milk cartons purchased enough for An to drink for a week?

Figure 4. Post-test


WORKSHEET 4 Name:..Class:........... ....

Match one figure and one product appropriately

3 x 6
6 x 3
5 x 4
4 x 5
4 x 2
2 x 4

Figure 3. The pre- test

Checking data to choose an appropriate analysis technique

- Subjects are independent. Each student fulfilled both the pretest and the posttest individually. Each student' worksheet was scored on a 10-point scale, and scores for one student did not affect scores for any other student.
- Each of the paired scores was obtained from the same student.
- The score difference data were tested on its normality by using the Shapiro-Wilk test. The paired differences are not normally distributed ($p < 0.05$).

Therefore, the non-parametric Wilcoxon signed-rank test was used for the statistical analysis. The effect size for the Wilcoxon signed-rank test was measured by a matched-pairs rank-biserial correlation and was computed by the simple difference formula (Kerby, 2014). The formula states that the matched-pairs rank-biserial correlation is equal to the difference between the favorable of and unfavorable evidence. For the Wilcoxon signed-rank test, the evidence consists of rank sums.

Findings

Self-developed models for grade 2 students learn multiplication of two natural numbers

Researchers mapped out self-developed models in grade 2 students' learning multiplication of two natural numbers, as shown in Figure 2. The rationale for Figure 2 includes:

- The philosophy of RME approach.
- The result of epistemological analysis of multiplication of two natural numbers showed that the multiplication of two natural numbers is the operation which allows, for two natural numbers a and b, the third natural number c equals to the sum of b terms a. That means $a \times b = a + a + \ldots + a$ (b terms).
- The idea of using the rectangular area model as a model-of arose from the fact that "the product $a \times b$, where a, b are natural numbers, can be considered as the rectangular area, where the length of the vertical straight side is a and the length of the horizontal straight side is b". Thus, a 5-column rectangular diagram with eight objects in each column can be used as a context for the multiplication $5 \times 8 = 40$.

The teaching process generated an environment in which students performed mathematization to gain mathematics knowledge

In Activity 1, students experienced mathematization through solving problems, which were solved by adding equal natural numbers together.

8. *The teacher: Xuan Trang! How did you know to take exactly the number of cakes?*
9. *Xuan Trang: I count two cakes for one plastic bag and take enough cakes for three plastic bags.*
10. *The teacher: So, how many cakes did you take?*
11. *Xuan Trang: Six, teacher!*
12. *The teacher: That means you count 2, 2, 2, right?*
13. *Xuan Trang: Yes.*

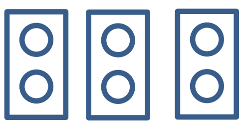
[...]18. The teacher: Khanh Nhi! How did you know to take exactly the number of cakes?

19. Khanh Nhi: Teacher, I remember one plastic bag is two cakes, I plus...I plus 2, 2 and 2.

[8-19] (M)

Above paragraph [8-19] (M) shows that two students experienced the same horizontal mathematization, but performed vertical mathematization in two different ways, as shown in Table 4.

Table 4. Mathematization in Task 1 of Activity 1 by two students

Horizontal mathematization	Vertical mathematization
Understand the problem by visualizing the figure of three plastic bags, two cakes in each bag.	Count 2, 4, 6
	Plus three numbers 2 together

After winning groups shared their solutions in two tasks of preparing presents, the teacher used the question-answer method to introduce a new concept instead of immediately introducing it, as shown in [22- 30] and [66-73] (M).

22. After commenting on the performance of the groups in Task 1, the teacher wrote on blackboard as follow.

"Task 1: 1 plastic bag has 2 cakes

Take enough cakes for 3 plastic bags!

(A blank line)

2 + 2 + 2 = 6"

23. Teacher pointed to "2 + 2 + 2 = 6" and asked the class: "Did we learn this?"

24. Students (in unison): Yes, we did.

25. Teacher: This is a multi-term addition. How about terms in this?

26. Students (in unison): The same.

27. Teacher: So, through the interpretation of the winning groups, what do we see?

28. Teacher (continue): With one plastic bag we will take two cakes, take enough for three bags, so that means… how many times are two cakes taken?

29. Students (in unison): three

30. Teacher: Right! That means what we have. Two is taken three times. Two here are two cakes (wrote "2 is taken 3 times" on the blank line) and the operation is 2 + 2 + 2 = 6.

[22-30] (M)

66. After commenting on the performance of the groups in Task 2, the teacher continued to write on blackboard as follow.

"Task 2: 1 plastic bag has 4 candies

Take enough candies for 5 plastic bag!

(A blank line)

4 + 4 + 4 + 4 + 4 = 20"

67. Teacher: Who can tell me? Four candies, or in short, how many times four are taken?

68. Student: It's five times.

69. Teacher: What is taken five times?

70. Student: Four is taken five times.

71. Teacher wrote "4 are taken 5 times" on the board, in the blank line.

72. Teacher: Here we see, two is taken three times, can be represented by addition like this (pointing to 2 + 2 + 2 = 6). But, there is another way to write "two is taken three times" by a new operation.

73. The teacher wrote "$2 \times 3 = 6$*" below the multi-term addition "2 + 2 + 2 = 6" and introduced "This is multiplication, and this sign (pointed to "*$\times$*") is the multiplication sign".*

[66-73] (M)

Similarly, the teacher asked one student to write the multiplication corresponding to the multi-term addition in Task 2 on the blackboard and the student wrote exactly.

Before introducing a new math concept, the teacher tried to help students switch from the context of "a is taken b times" into operations, add b times the number a, where a and b are the specific numbers in Task 1 and Task 2. As we see in [8-19] (M) and [22-73] (M), the teacher did several actions as commenting on groups' performance, interviewing the winning groups, re-interpreting the shares, asking and answering questions, and writing the main contents on the blackboard.

The Activity 4 was implemented for students work individually, in group and all class, with ten sub-activities, as shown in the following Table 5.

From Activity 4d to 4j, students did not work directly on the math-link cubes but they observed the rectangles formed on the presentation slides. Images or movements being presented on slides should simulate students' actions when they directly work on math-link cubes. In the period students perform Activity 4f and 4g [11-62] (3TT), "a block of 3 appears on the right side of the screen and starts to move to the left and stops" can stimulate students' actions "drag a block of 3 from right to left". Next, "another block of 3 appears on the right and moves on the left until it is next to the first block of 3 then stops". And so on until the tenth block of 3. Through observing the screen, students took turns writing the corresponding multiplications according to the rhythm of presentation slides. As a result, students made the 3 Times Table by themselves. The progressive mathematization taking place in activities from 4d to 4j are also completely similar to those in activities 4a, 4b and 4c, so authors do not show them in detail. The following Table 6 showed that implemented Activity 4a brought all students up to seven opportunities to participate in mathematization processes.

Table 5. Implementing Activity 4 in class 2A and students' learning outcomes

Students' activity	Type of activity	(Code) Task	Paragraph (Record)	Students' learning outcomes
Activity 4: Write the corresponding multiplication with a rectangle formed by math-link cubes.	Individual (Use math-link cubes)	**(4a)** Make a rectangle, then write the corresponding multiplication (8 multiplications asked)	[15-84] (F-P) [1-13] (2TT)	Only 5/46 students failed in only one multiplication. Most students made math-link rectangles and wrote exactly corresponding multiplications.
	Teamwork (Use math-link cubes)	**(4b)** Make 2 rectangles according to 2 patterns, which are traces of 3 x 4 and 2 x 6 rectangles. Then, fill in the blanks with the total number of math-link cubes were used	[1-13] (5TT)	All groups assembled 2 rectangles according to patterns and wrote the correct number. Students worked in the group quite well and knew how to coordinate in order to complete tasks.
	Teamwork (Use math-link cubes)	**(4c)** Make The 5 Times Table	[14-28] (5TT)	All groups made exactly The 5 Times Table. Students initially learned how to coordinate, assign tasks to perform the group's tasks.
	All class	**(4d)** Read The 2 Times Table while observing images of blocks of 2	[1-2] (3TT)	All students read in unison loudly and exactly.
	Individual	**(4e)** Answer the questions "How many eggs in the tray?" after observing images of rectangle trays of eggs 2x3, 2x6 and 2x5	[3-9] (3TT)	Three students called and they gave the correct answer quickly
	Individual	**(4f)** Observe rectangular area models and make the 3 Times Table	[11-57] (3TT)	Most students wrote exactly the 3 Times Table.
	All class	**(4g)** Read the 3 Times Table while observing images of blocks of 3 on presentation slides	[60-62] (3TT) [1-2] (4TT)	All students read in unison loudly and exactly the 3 Times Table.
	Individual	**(4h)** Read multiplication corresponding to images of rectangles	[14-42] (4TT)	9 students called, where 6 students read exactly the multiplication.
	All class	**(4i)** Read the 4 Times Table by observing images of blocks of 4 on presentation slides	[43- 66] (4TT)	All students read in unison loudly and exactly
	Individual	**(4j)**Write multiplications corresponding to images (rectangle 4x6 tray of eggs, beverage) being shown on slides	[86-90] [108-117] (4TT)	Only 4/46 students (8.7%) failed

Table 6. Mathematization processes students performed in Activity 4a

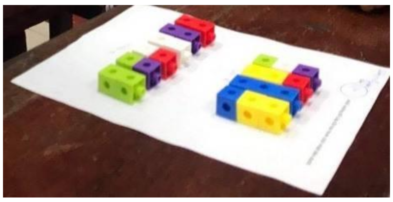
Paragraph (Record)	Students' action	Mathematization processes: Visual image (when students' actions are done) →	Meaning →	Mathematics
[15-26] (F-P)	Put 3 blocks of 2 next to each other		2 is taken 3 times	2 x 3 = 6
[46-61] (F-P)	Put 2 blocks of 5 next to each other		5 is taken 2 times	5 x 2 = 10
[63-66] (F-P)	Put 4 blocks of 2 next to each other		2 is taken 4 times	2 x 4 = 8
[67-75] (F-P)	Put 1 block of 5		5 is taken 1 time	5 x 1 = 5
[79-84] (F-P)	Put 3 blocks of 1 next to each other		1 is taken 3 times	1 x 3 = 3
[4-9] (2TT)	Put 4 blocks of 2 next to each other		2 is taken 4 times	2 x 4 = 8
[12-13] (2TT)	Put 1 block of 2		2 is taken 1 time	2 x 1 = 2

Indeed, throughout directly working with math-link cubes, each student got a visual image. It helped him realize that it is a rectangle formed by the action "a is taken b times", where a is the total number of math-link cubes in one block - one column of the rectangle and b is the number of column of the rectangle. He then wrote the multiplication of two natural numbers. Next, he found its product by counting the total number of all math-link cubes used to make the rectangle.

For instance, as we observed from a close-up video of two students sitting at the same table in activity 4a, after making a rectangle formed by four columns (each column is a block of 2 cubes), they wrote the number 2 on the paper and paused to count the number of columns. They got 4, so they continued to write the sign "×" and the number 4. Again, they paused to count all cubes in the rectangle. They got 8, so they wrote the sign "=" and the number 8. Finally, they had the multiplication "$2 \times 4 = 8$". It can be said that these students did experience mathematization processes with the multiplication "$2 \times 4 = 8$", hence, they clearly understood what they wrote down.

Similarly, mathematization processes took place in Activity 1b are illustrated in Table 7.

Table 7. Mathematization processes students performed in Activity 4b

[Paragraph] (Record)	Mathematization processes: Visual image (when students' actions are done) →	Meaning →	Mathematics
[1-13] (5TT)		3 is taken 4 times	3 x 4 = 12
		2 is taken 6 times	2 x 6 = 12
		24 cubes be used	12 + 12 = 24

Students worked in group of 2 students. Firstly, students used math-link cubes to make two rectangles according to two traces of rectangles 3 x 4 and 2 x 6. Then, they identified how many math-link cubes they used. In this activity, students found two solutions to find the answer: counting the total number of cubes on two rectangles, or adding two products.

In Activity 4c, students worked in a group to make the 5 Times Table. The teacher gave 50 math-link cubes and one notepaper to each group. The results of the observation showed that students were still awkward at first. However, under the teacher's guidance, groups initially learned how to coordinate group's tasks and assign tasks to perform. After finishing Activity 4c, they had the 5 Times Table on the notepaper. Figure 5, as shown below, includes some images we took from a recorded video of one group's performance in Activity 4c.

Figure 5. Teamwork in Activity 4c

The following paragraph is written from a recorded video of one group's working process in Activity 4c.

24.1 *Teacher: Where are your ten numbers 5?*

24.2 *All members in the group: Here they are (each student was holding some blocks of 5)*

24.3 *Teacher: Now, let's put them together!*

24.4 *One student put all blocks of 5 together.*

24.5 *Teacher: Ok, let's begin to make the 5 Times Table!*

24.6 *Teacher: Make five times one, how?*

24.7 *One student: five times one, take one (while he was holding up one block of 5 cubes)*

24.8 *Teacher: Right, put one number five on the notepaper, so we have five times one equals?*

24.9 *All students in a group (in unison): Equal five!*

24.10 *Teacher: Whose turn is it to write?*

24.11 *Some students appointed the student he was standing next to Hoa, on the right.*

24.12 *Appointed student wrote on the notepaper:* $5 \times 1 = 5$

24.13 *Teacher: Is it exactly?*

24.14 *All students in unison: Yes, exactly!*

24.15 *Teacher: Go ahead! Pass the notepaper to your right hand. It's your turn to put cubes.*

24.16 *This student quickly passed the notepaper to a friend on his right hand (but he seems not to understand how to take cubes)*

24.17 *Teacher: Take one more block of 5, then place it next to the already block.*

24.18 *The student did take one block of 5 and put it next to the already block.*

24.19 *Teacher: One of you, let's write the multiplication now we have*

24.20 *The student (in turn) still hesitated, put pen on the notepaper but not written yet.*

24.21 *Another student read the multiplication for his friend could write:* $5 \times 2 = 10$

24.22 *Teacher: Right, so on, one takes cubes, one writes the multiplication!*

24.23 *The student quickly passed the notepaper to a friend on the right hand and takes one block of 5 and put it next to two already blocks.*

24.24 *Teacher: Let's write the multiplication!*

24.25 *Student wrote* $5 \times 3 = 15$.

24.26 *Teacher: Why do five times three equal fifteen?*

24.27 *Student (who just wrote 5* x *3 = 15): Because ten plus five equals fifteen.*

24.28 *Teacher: Good job; each turn, we put one more block of five, we plus five. Go ahead!*

24.29 *The group took turns in putting blocks of 5 and writing the multiplication from* 5×4 *to* 5×8, *respectively.*

24.30 *Teacher: So, how many times we took the number 5? (after the ninth block of 5 was put on the notepaper)*

24.31 *Student (so quickly): nine times.*

24.32 *Another student: nine times (after counting all blocks on the notepaper)*

24.33 *Students continued to do tasks.*

24.34 *Teacher: So, how many times we took the number 5? (after the tenth block of 5 was put on the notepaper)*

24.35 *Student: five times ten*

24.36 *Teacher: How many times we took the number 5?*

24.37 *Students in unison: ten times.*

[24.1-24.36] (5TT)

Paragraph [24.1-24.36] (5TT) shows that students understood the multiplication of two natural numbers. They realized that $5 \times n$ means the number 5 is taken n times, and could calculate its product by adding 5 for each taking turn. Thus, all students have chances of mathematization if they actively participate in the group's task situated in Activity 4c.

In Activity 5, students must identify or represent a rectangle for a given multiplication. Activity 5 was organized at the end of the lesson on Factor-Product and the lesson on The 2 Times Table, with three particular activities: 5a, 5b, and 5c (see Table 8). It is designed for the objective of building a "model-of" in students' minds. This model helped students clearly understand the multiplication of two natural numbers. There were only 16/46 students correctly colored above three rectangles in Activity 5a. Some students only colored enough the number of unit squares corresponding to the product, but all colored unit squares could not make a rectangle. After working in Activity 5b, the percentage of students' success in Activity 5c (correctly matched ten rectangles with ten multiplications) was much higher than the success rate in Activity 5a, with 35/46 students, accounting for 76.1%.

Table 8. Implementing Activity 5 in class 2A and students' learning outcomes

Students' activity	Type of activity	(Code) Task	Paragraph (Record)	Students' learning outcomes
Activity 5: Determine or represent a rectangle for a given multiplication	Individual (use Worksheet)	**(5a)** Color the rectangle corresponding to a given product.	[85-90] (F-P)	16/46 students (34.6%) met above 60% requirements.
	Individual (Use math-link cubes)	**(5b)** Make math-link rectangles corresponding to the multiplication given on the blackboard. Then, write the 2 Times Table	[14-87] (2TT)	Most students made rectangles from math-link cubes and wrote exactly the 2 Times Table.
	Individual (use Worksheet)	**(5c)** Match a figure in column A with multiplication in column B, and fill in the blanks with the appropriate number (10 figures are 10 rectangles corresponding to 10 multiplications in the 2 Times Table)	[118-120] (2TT)	35/46 students (76.1%) matched exactly 10 figures with 10 multiplications.

The teaching process provided a positive and enjoyable learning environment, in which students reached the understanding of the multiplication of two natural numbers and developed positive attitudes towards learning math

Impact on students' attitude towards learning

In the proposed teaching process, researchers used colorful math-link cubes as learning resources. These interesting learning resources made students feel as though they were playing with toys. In other words, students felt at home with numbers and operations. As a result, they felt more excited about learning activities. By observing six implementing lessons, the study found that students showed enthusiasm and actively participated in activities organized in class. It is concluded from the following manifestations as focused, paid attention to fulfilling requirements, actively raised their hands to volunteer to answer the question, students' smiling faces when completing the task, actively participated in group activities, as shown in Figure 6.

Figure 6. Students of class 2A in learning activities

Impact on learning outcomes

In a quantitative analysis: The students' responses on pre-post tests were collected. The obtained data were analyzed with the help of IBM SPSS Statistic Program 28, as shown in Table 9, Table 10 and Table 11.

Table 9. Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum	Percentiles		
						25th	50th (Median)	75th
post	44	7.1216	3.3408	.00	10.00	4.1650	6.6700	10.0000
pre	44	3.4857	3.8018	.00	10.00	.0000	1.6650	6.6700

Table 10. Ranks

		N	Mean Rank	Sum of Ranks
post - pre	Negative Ranks	6[a]	15.83	95.00
	Positive Ranks	30[b]	19.03	571.00
	Ties	8[c]		
	Total	44		

a. post < pre
b. post > pre
c. post = pre

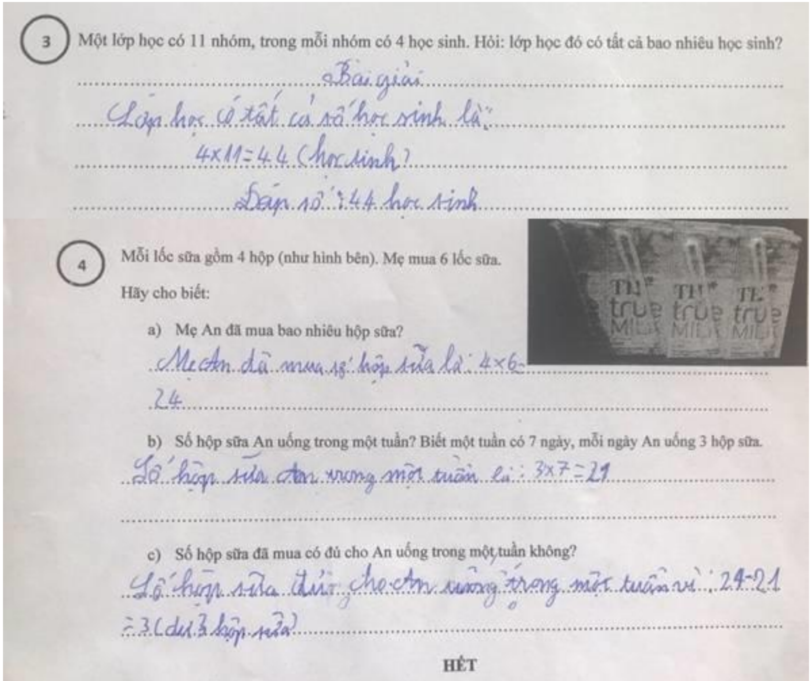
Table 11. Wilcoxon Signed Ranks Test

	post - pre
Z	-3.785*
Asymp. Sig. (2-tailed)	<.001
Exact Sig. (2-tailed)	<.001
Exact Sig. (1-tailed)	<.001
Point Probability	.000

*. Based on negative ranks.

A Wilcoxon signed-rank test showed that there was a significant difference ($Z = -3.875, p < 0.001$) between scores given for the pre-test compared to the post-test. The median score for the posttest was 6.67 compared to 1.66 for the pretest. In addition, the effect size $r = 0.7$ suggested a high practical significance. In other words, the teaching process helped grade 2 students understand the multiplication of two natural numbers.

In a qualitative analysis: The research findings showed that students actively participated in several activities in six experimental lessons. Mathematization processes, which took place throughout these activities, were illustrated in Table 4, Table 6 and Table 7. Students experienced mathematization processes from taking a number a unit cubes (in a block, repeat b times) to make a rectangle, and then realized the multiplication of two natural numbers (a, b) equals c, where c is the sum of b times terms a. After analyzing the two mathematization processes (in Activity 4a and Activity 4c), we can see that the rectangular area model played a significant role in helping students approach formal knowledge. In this case, formal knowledge is the concept of the multiplication of two natural numbers. Thus, most of the students in the class understood the mathematics knowledge and could solve the problem involving the actual meaning of the multiplication, as illustrated by studentA6's worksheet (Figure 7).



3 Một lớp học có 11 nhóm, trong mỗi nhóm có 4 học sinh. Hỏi: lớp học đó có tất cả bao nhiêu học sinh?

Bài giải

Lớp học có tất cả số học sinh là:

4×11=44 (học sinh)

Đáp số: 44 học sinh

4 Mỗi lốc sữa gồm 4 hộp (như hình bên). Mẹ mua 6 lốc sữa.

Hãy cho biết:

a) Mẹ An đã mua bao nhiêu hộp sữa?

Mẹ An đã mua số hộp sữa là: 4×6= 24

b) Số hộp sữa An uống trong một tuần? Biết một tuần có 7 ngày, mỗi ngày An uống 3 hộp sữa.

Số hộp sữa An uống trong một tuần là: 3×7=21

c) Số hộp sữa đã mua có đủ cho An uống trong một tuần không?

Số hộp sữa đủ cho An uống trong một tuần vì: 24-21 =3 (dư 3 hộp sữa)

HẾT

Figure 7. Student A6's worksheet

(The translation of the Figure 7: 3) A class has 11 groups, in each group there are 4 students. How many students are there in the class? Solution: The number students in the class are $4 \times 11 = 44$ (students). 4a). How many cartons of milk did An's mother buy? An's mother bought the number of cartons of milk: $4 \times 6 = 24$, b) How many cartons of milk An drink in a week? Know that: there are 7 days in a week, and An drinks 3 cartons of milk every day. The number of cartons of milk An drank in a week were $3 \times 7 = 21$ c) Is the number of milk cartons purchased enough for An to drink for a week? The number of milk cartons is enough for An drinks in a week because $24 - 21 = 3$ (3 cartons left over)).

Discussion

Many teachers have paid attention to teaching methods to help students learn actively. However, if assessed seriously and intrinsically, most of them are just positive outward manifestations. Students work to perform tasks in a stereotypical, mechanical way, not actively perform cognitive activities. Therefore, to form and develop active learning in learners, teachers need to focus on the following principles: 1- Create learning motivation and interest in learners; 2- Get learners involved; 3- Motivate and Guide learners.

These positive results of the study showed that the teaching process underpinned the RME approach met the three above principles. It engaged learners in action and thinking about what they were doing. It can be said that mathematization is a powerful tool, a key strategy teaching to form and develop active learning in learners. As a result, students develop their understanding of mathematics and their attitude towards math learning. This statement is consistent with many studies on the effectiveness of the RME approach. For instance, Drijvers et al. (2019) concluded that the RME approach positively affected students' mathematical thinking and problem-solving abilities. Fauzan et al. (2020) also found that RME made math learning meaningful and improved students' reasoning. The study results are consistent with Laurens et al. (2018) and Syafriafdi et al. (2019): students from RME project classes are more likely to get the correct answer and more likely to approach the questions according to the way they understand the problem.

In this study, the teaching math concepts process begins with solving contextual problems. After that, the class teacher built two models for students to approach and understand a new concept. This approach is in line with Anh and Cuong (2020). They also suggested it as one solution to apply RME in teaching math. The authors also agreed with their note about models used in RME. These models are intrinsically different from the mathematical model or mathematical modelling. In the case of teaching the multiplication of two natural numbers, each rectangle formed by unit cubes plays as a model-of, while the action "a is taken b times" plays as a model-for. These realistic, vivid visual models have helped students reinvent knowledge effectively.

The research emphasized the role of the teacher in using mathematization as a pedagogical tool. Throughout the teaching process, the teacher observed students' learning activities, provided timely support to individuals or groups, commented and guided them to reinvent knowledge. RME can be effectively applied and deployed if and only if the classroom teacher has an awareness of the effectiveness of RME approach and a fundamental understanding RME. Only the classroom teacher can perform a function that organizes activities towards the target of the HLT, makes students feel free to employ their mathematization abilities (Wheeler, 2001). The classroom teacher plays a prime role in generating a collaborative, problem-solving environment (Cobb, 1999) and supporting learners (Wahyudi et al., 2017).

The findings also showed that although the chances for each student to experience mathematizing processes are the same, the level of success depends on student's individual ability, especially the ability to perform thinking activities in the vertical mathematizing. This is in line with the study on the students' fluency, flexibility in solving problems: the high-ability students were the most fluent and flexible, meanwhile the low-ability students find difficult to understanding the problems and made many errors in solving (Arifin et al., 2021). Grade 2 students should have a basic background such as experience in counting, the ability of recognizing the force of a set, the symbols of natural numbers in order to be successful in the vertical mathematizing. These foundations should be built for them by both their parents at home and their teachers at nursery school.

Conclusion

It is concluded that mathematization is the key teaching strategy to develop learners' competencies. Through working with realistic contexts, grade 2 students could perform thinking activities successfully. As a result, they could experience mathematization processes, especially vertical mathematization. The research results showed that grade 2 students achieved a comprehensive understanding of the multiplication of two natural numbers. Thus, they could solve problems involving the practical meaning of the multiplication of two natural numbers. In addition, the study highlighted the role of interesting learning resources as realistic, vivid visual contexts for primary students. Along a teaching process, which was well-designed and met the fundamental principles of RME, using colorful math-link cubes generated a positive and enjoyable learning environment. Students actively participated in classroom activities. They found it enjoyable to solve problems in both ways: individually and collectively. Their joy of learning led them to be successful in guided reinvention and so interested in math learning. Besides, the study contributes to introducing RME as an effective approach for teaching math in the orientation of developing learners' competencies. It helps teachers clearly see the basic ideas that need to be ensured when teaching math in general and teaching the multiplication of two natural numbers for elementary students in particular.

Recommendations

There are two most important recommendations for practitioners and future researchers who need to study and apply RME to improve the quality of teaching math. Firstly, map out the Hypothesis Learning Trajectory based on an epistemological analysis of the knowledge and three fundamental principles of RME. Secondly, make the classroom teacher aware of the philosophy of RME.

For further research, "how to communicate and organize learning activities so that teachers can support students as much as possible" and "how to make teachers aware of RME" are ideas that researchers can continue to implement.

Limitations

The teaching experiment was conducted on only one class, with 46 students. There was lack of a survey on students' attitudes towards learning mathematics and a formal interview about teachers' perceptions after the experiment.

Acknowledgements

Authors would like to express deep gratitude to Master Tran Quoc Toan, a math teacher at the Pedagogy Practice School, for his valuable comments on the process of teaching the multiplication of two natural numbers to grade 2 students. Also thank to 46 students of the class 2A of Pedagogical Practice School in school year 2020-2021, especially for the enthusiastic help of the teacher Tran Ngoc Hai Anh. She worked hard to gain a fundamental understanding of RME to organize activities towards the target of the process, guide students if students need. Besides, the researchers also thank to the teacher Le Van Hieu for his assistance in shooting video clips for all experimental lessons.

Authorship Contribution Statement

Hong Duyen: Design, writing, editing, review, data acquisition, data analysis. Loc: Conceptualization, review, supervision, final approval.

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