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Problem-Solving Process of Students with a Reflective Cognitive Style Based on the Action-Process-Object-Schema Theory

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Abstract: The skill to solve mathematical problems facilitates students to develop their basic skills to solve problems in daily life. This study analyzes students' problem-solving process with a reflective cognitive style in constructing probability problems using action, process, object, and schema theory (APOS). The explanatory method was used in this qualitative study. The participants were mathematics students at the Department of Mathematics, Universitas Negeri Semarang. The researchers collected the data with the cognitive style test using the Matching Familiar Figure Test (MFFT), used a valid problem-solving skill test, and the interview questions. The data analysis techniques used were processing and preparing the data for analysis, extensive reading of the data, coding all data, applying the coding process, describing the data, and interpreting the data. The results showed that (1) the problem-solving process of students with symbolic representation was characterized by the use of mathematical symbols to support the problem-solving process in the problem representation phase; (2) the problem-solving process of students with symbolic-visual representation was characterized by the use of symbols, notations, numbers, and visual representation in the form of diagrams in the problem representation phase.

Keywords: *APOS theory, cognitive style, problem-solving, reflective.*

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Introduction

Problem-solving skill is an essential element of the study of mathematics (Drijvers et al., 2019; Sudarsono et al., 2022). The skill is the mathematical core to achieving the targeted learning goal (Branca, 1980; Gök & Sýlay, 2010; Surya et al., 2017). Problem-solving is an inseparable and integral part of learning mathematics and solving problems (National Council of Teachers of Mathematics, 1989; Rahayu & Kartono, 2014). The skill to solve mathematical problems facilitates students' development of cognitive skills and life skills to solve problems in daily life (Suarsana et al., 2019).

Problem-solving is the cornerstone of mathematics education. Therefore, mathematics teacher candidates must master problem-solving skills (Avcu & Avcu, 2010). For teacher candidates, excellent mathematics problem-solving skills, understanding, and strategy facilitate the development of their future service program. Unfortunately, most teacher candidates still had problems understanding the related questions, proper problem-solving strategy, and non-routine problem-solving matters (Güner & Erbay, 2021). These gaps caused teacher candidates to encounter problems during their practicum, especially in teaching problem-solving skills.

Students' problem-solving skills can be observed in their problem-solving processes (Docktor et al., 2016; Selçuk et al., 2008). The problem-solving process is a conceptual-psychomotor organizational process to create a new pattern. Therefore, instructors need to understand students' cognitive processes to determine students' positions and commit errors in problem-solving (Bintoro et al., 2021).

When solving mathematical problems, individuals usually apply several stages to solve them. Polya suggests a four-stage problem-solving process: (a) understanding the problem, (b) developing a plan, (c) executing the plan, and (d) looking back (Polya, 1971). According to Gravemeijer et al. (2017), the problem-solving stages include: Identifying where mathematics is applicable, translating the practical problem into mathematical problems, solving the mathematical

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problems, and interpreting and evaluating the results. Pujiastuti et al. (2018) explain five steps of problem-solving: reading and understanding, organizing the strategy, solving the problem, confirming the process, and confirming the answer. Although theories of the problem-solving process are widely used, the process of problem representation has not yet been discussed.

In this study, mathematics students' problem-solving process consisted of five stages. They consisted of reading and understanding the problem, presenting it, developing problem-solving strategies, solving the problems, and confirming the answer. Table 1 shows the indicators for each problem-solving stage.

Table 1. The Problem-Solving Process Indicators

Stages	Indicators
Reading and understanding the problem.	1. Reading the questions 2. Being capable of writing the given matters of the questions correctly. 3. Being capable of explaining the given information
Representing the problem	1. Being capable of writing problem-solving questions 2. Being capable of representing the problem through mathematics models, figures, algebraic symbols, diagrams, graphs, and scripts
Constructing problem-solving strategies	1. Being capable of writing and systematizing the applied formula correctly 2. Being capable of writing the strategy sequences in solving problems orderly and entirely by using the given elements, the pre-made mathematics model, and the presented problem representation
Solving problem	1. Being capable of promoting mathematics calculation accurately based on the formula, strategies, and the chosen representations 2. Being capable of determining the final answers based on the questions
Confirming the answer	1. Being capable of stating the answers to question interpretations

The difference between the problem-solving stages and earlier research is in the problem statement stage. Students formulate the goals of the questions and choose the correct representation according to the questions asked. Problem representation aims for students to represent the problem in pictures, tables, graphs, symbols, notations, diagrams, equations, or mathematical expressions to help develop problem-solving strategies.

Problem-solving is a part of a cognitive process for solving problems (Marzano et al., 1988). A person's thinking process can be tracked in several ways. One is to track the thinking process using action, process, object, and schema (APOS) theory. APOS theory identifies student mathematical cognition as the tendency to respond to problems in mathematical situations. In this case, the student reflects on the problems and solutions in the social context, the constructions and reconstructions of actions, processes, and objects, and schema management to deal with the problems (E. D. Dubinsky & Mcdonald, 2001). APOS theory involves a mechanism of mental interiorization, coordination, reversal, encapsulation, de-encapsulation, and thematization (Arnon et al., 2014). The mental structure of APOS illustrates an individual cognitive structure in solving mathematical problems (Olesova & Borisova, 2016). The problem-solving process of mathematics problems can be realized and constructed with the mental structure of APOS (Hidayatullah, 2019; Inglis, 2015; Moll et al., 2016; Sutarto et al., 2018).

Previous research has conducted various APOS analyses in mathematical studies. Borji et al. (2018) combined APOS and OSA theories to analyze students' understanding of functional and differential graphs. Inglis (2015) discusses the importance of APOS theory in learning arithmetic. Sumaji et al. (2020) studied junior high school students' communication process in solving problems based on the APOS theory framework. The weakness of this previous study was that it only examined the mental structure of actions, processes, objects, and schemas constructed by students. It is still rare to look more closely at the characteristics of teacher candidates at the stages of the problem-solving process in constructing mental structures and mental mechanisms, especially probability material.

Students' problems are related to the complexity of mathematical problems and students' cognitive styles (Junarti et al., 2020). Cognitive style refers to a particular method of understanding, remembering, thinking, solving a problem, and responding to information or different environmental situations (Altun & Cakan, 2006; Colbert et al., 2008; Ifelunni et al., 2022; Kozhevnikov, 2007). Cognitive style is closely related to a person's learning style and becomes an individual uniqueness. Cognitive style includes reflective and impulsive dimensions (Kagan, 1965). These aspects refer to an individual's tendency to respond immediately to problems with uncertain answers (Rozencwajg & Corroyer, 2005; Wulandari et al., 2020). Students with a reflective cognitive style respond slowly but carefully and tend to find the correct answer. Previous research has shown that reflective individuals have better metamemory than impulsive individuals (Michalska & Zajac, 2015) and have better critical thinking skills than impulsive individuals (Cahyono et al., 2019). The results of this study have the strength to show the characteristics and abilities of reflective and impulsive individuals in mathematics. However, it still has drawbacks, namely the lack of analysis of thinking processes, especially problem solving by prospective teachers.

Researchers analyzed the problem-solving process to determine the probability question based on probability problems and individual cognitive characteristics. Problem-solving is a cognitive process in which students' unidentified solution is the goal (Atteh et al., 2017; Peng et al., 2020). The cognitive style is a valuable reference for solving probability problems. The cognitive styles, especially the reflective and impulsive ones, contribute critically to problem-solving. The cognitive styles enable students with different cognition to solve different problems (Cataloglu & Ates, 2014).

From the initial observations, it appears that the prospective teachers still have difficulties solving the problems of equal opportunities. The problems students face with probability material are quite complex. Many previous studies reported that mastery of probability is far from average (Estrada et al., 2018). Misconceptions about probability cause difficulties in mastering probability (Masel et al., 2015; Triliana & Asih, 2019), conceptual difficulties in solving problems (Sezgin-Memnun et al., 2019), procedural difficulties in performing manipulations or algorithms (Arum et al., 2018), and interpretive difficulties, i.e., not being able to interpret probability problems correctly (Konold, 2017; Pisarenko, 2018). Based on previous research, researchers have not found a theory that discusses the problem-solving process for probability material using APOS theory.

This research analyzed students' problem-solving processes based on reflective cognitive style. This study analyzed students' problem-solving processes with a reflective cognitive style in constructing probability problems. The applied results improved the learning materials for students on probability problems based on the cognitive style.

Methodology

Research Design

This qualitative research used the explanatory method. The researchers followed the problem-solving process by asking students problem-solving questions. The researchers did this until they found the most appropriate participants for the research objectives. After exploring the issues, the researchers included participants with symbolic and visual representations.

Research Goal

This research analyzed the problem-solving process of reflective cognitive students in solving probability problems using APOS theory. Specifically, students' problem-solving skills with symbolic and visual representation were investigated. Then, the researchers used APOS theory to reveal students' problem-solving processes.

The Study Group and Data Collection

The research was conducted at the Department of Mathematics, Universitas Negeri Semarang, and the data were collected from second-semester teacher candidates. The study included 46 participants, fourteen males and thirty-two females aged 17-20. The researchers used a purposive sampling technique to identify the participants. Participants were administered a Matching Familiar Figure Test (MFFT) to collect data. To determine which individuals have reflective, impulsive, slow-accurate, and fast-accurate cognitive styles, the Matching Familiar Figures Test (MFFT) instrument was used, which was developed by Warli (2013) and consisted of 13 items. The MFFT includes a visual comparison task that requires respondents to select an image that matches the standard from several alternative images. The MFFT test requires the respondent to perform analytical processing to select the correct image.

Based on the results of the MFFT, the researchers divided the students into four categories: impulsive, reflective, fast-accurate, and slow-inaccurate. Based on the test, 16 students exhibited a reflective cognitive style. Later, the researchers administered a test of problem-solving skills using a probability material of Bayes' theorem. Based on the variation of cognitive processes in problem-solving, the researchers used the uniqueness of representations and communication skills. The researchers interviewed six out of sixteen students with reflective cognitive styles. Then, the researchers selected two participants out of the six to be further interviewed about their problem-solving processes. The researchers selected these two participants because they applied the five stages of problem-solving, specifically the representation of the subject's cognitive process based on APOS theory. The participant with symbolic representation was named S1, and the participant with symbolic-visual representation was named S2.

The problem-solving test question is a research instrument used to collect data on the thinking process involved in solving problems. It consisted of one question. The problem was developed from problem-solving indicators to probability material. *"About 1% of women aged 40-50 are at risk of developing breast cancer. Mammography, or mammogram, is a scan test to see an image of the breast gland and surrounding tissue. If a woman at risk for cancer undergoes a mammogram, the chance of a positive test result is 90%. If a woman at no risk of cancer undergoes a mammogram, the chance of a positive test result is 10%. If a woman's mammography test is positive, what is the chance that the woman has breast cancer?"*

Documentation data in this study were available in the form of written test responses and videos as respondents answered problem-solving questions. The interview questions were developed using the problem-solving process, mental structure, and mental mechanism APOS indicators. For example, to investigate the students' problem-solving process in the reading stage and to understand the problem, an interview was conducted on the mental structure of action and the mental mechanism of interiorization with the following questions: (a) What did you do after you received the questions? (b) How many times did you read the questions? (c) What information did you get from the questions? Can you tell me? Based on the test answer sheets, in-depth interview questions were used to confirm the mental structures and mechanisms in the problem-solving process. The interviews were also used to support the data coding process. These data were obtained by recording the Zoom session during the interview. Field notes were used as research notes during the research process.

An expert validated the test instrument for solving probability problems. Based on the expert feedback, it was determined that the test instrument was appropriate to measure the problem-solving process. Then, a pilot study was conducted to obtain empirical data. The results showed that the problem-solving test instrument was valid and reliable. The interview guide used to reveal in-depth information was also validated by experts.

Analyzing of Data

The researchers used triangulation with multiple data sources to ensure validity and reliability. The researchers compared the different data sources from the problem-solving tests and the interviews to answer the research question. This student's thinking process was analyzed using Action-Process-Object-Schema (APOS) theory (Arnon et al., 2014; Bintoro et al., 2021).

The researchers qualitatively analyzed the data obtained. The researchers gave students written tests on probability and in-depth interviews to identify the stages of procedural data analysis used for problem-solving: (a) processing and preparing the data for analysis, (b) reading data extensively, (c) coding all data, (d) applying the coding process to describe the reflective students' problem-solving processes, (e) describing the problem-solving process using APOS theory, and (f) interpreting (Cresswell, 2014). The researchers analyzed the responses to the problem-solving tests and the results of the in-depth interviews during these phases to examine students' problem-solving processes with symbolic and visual representation.

Findings / Results

The students' problem-solving processes based on APOS theory are presented in this section.

The S1's Problem-Solving Process with Symbolic Representation Type

S1 read and understood the problem. This stage indicated reading the question and writing the given elements in the questions. Once S1 received the questions, S1 read the questions directly. The mental structure of S1's actions was evident from the interview results.

R: What action did you take once you received the questions?

S1: I read the questions.

R: How many times did you read the questions?

S1: I read the questions twice to understand and obtain information.

From the written responses and interview results, S1 was fluent and correctly identified the information and problem components. S1's internalization included the likelihood of women having cancer, the likelihood of women having positive test results when they have cancer, and the likelihood of women having a positive test when they do not. Figure 1 shows the result of interiorization the written task of S1.

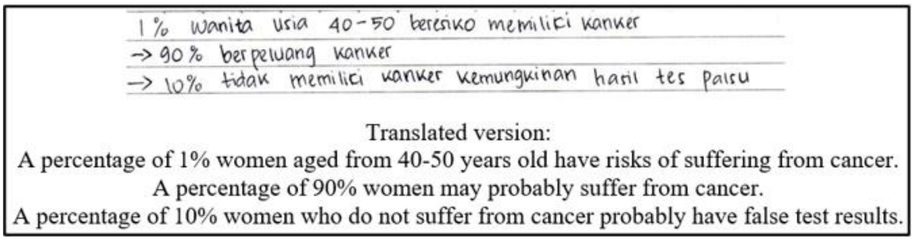


Figure 1. The Results of S1's Interiorization in the Form of a Written Task by Identifying the Problem Components

S1 could identify the given informative components and integrate the perception and new knowledge into a schema in his mind. The following quotes show the results with S1 about interiorization mental mechanisms.

R: What information did you get from the questions? Can you tell me?

S1: The information is about a percentage of 1% of women aged 40 to 50 who have breast cancer. Then, the remaining percentage, 90% of women, have positive test results if they have cancer. Then, the remaining 10% percent that does not suffer from cancer will probably have false test results.

S1 could correctly explain the given information about the problems based on the interview.

The second step in the problem-solving process was to state the problem. S1 was able to write and reproduce the issue in question accurately. The findings about how S1 determined the goal of problem-solving are as follows.

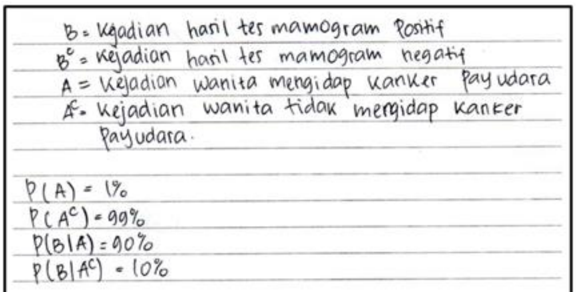
R: What is being questioned in the questions?

S1: The probability of women who have cancer if the mammogram test results are positive.

P: In your opinion, is the given information already enough to solve the problem?

S1: Yes, it is.

As for the structure of the mental process, S1 coordinates some interiorized components of the problem. Figure 2 shows the coordination of S1 in the form of a mathematical symbol representation.



Translated version:
B refers to the positive mammogram test result.
B^c refers to negative mammogram test result.
A refers to women suffering from breast cancer.
A^c refers to women who do not suffer from cancer.

Figure 2. The Coordination Result of S1

The researchers interviewed the participant to determine the mechanism of mental coordination. S1's interview results on promoted coordination can be found below.

R: What did you think after you received the problem components?

S1: I made an analogy for each event. Then, I sought any complement to the event. I created an analogy of B as the event of a positive mammogram test result. Then, I wrote B^c after a negative mammogram test result. I wrote A as the analogy of women who have breast cancer. Then, I wrote A^c as the analogy of women that did not have breast cancer.

R: What was the next stage you did?

S1: I created an analogy of data from the questions.

R: Why did you choose these representations to present problems?

S1: I did so to facilitate me in writing the data and using the formula.

Figure 2 and the interview result show how S1 promoted the mental coordination mechanism by analogizing the events and the previous events as complements. Then S1 converted them into probability values based on the given information. The figure shows that S1 converted the questions into symbols to facilitate the identification of the given elements and replace the values in the applied formula. This process shows that S1 was able to present the problem with an accurate symbolic representation.

S1 promoted the reversal of mental mechanisms in constructing a problem-solving strategy. This mechanism was an attempt to retrieve the concept of conditional probability. S1 coordinated the concept with the interiorized components of the problems. Figure 3 shows S1's written reversal of mental mechanisms in developing the problem-solving strategy.

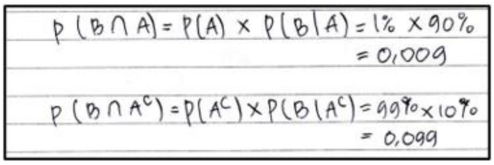


Figure 3. S1's Reversal Results in Constructing Problem-Solving Strategy

The interview result of S1's reversal mental mechanism process is given below.

R: What was your strategy to solve the problem?

S1: I used the Bayes theorem to answer the questions. However, I tried to determine the possible values of two overlapping events based on the concept of conditional probability. I included the result in my calculation using the formula of Bayes' theorem.

The figure and the interview result show S1's mental process indicated by the reversal of mental mechanisms. S1 did this by recalling the concept of conditional probability. S1 wrote conditional probability formulas to determine all internalized events, including symbolic representations and probability value determinations. This process shows that the strategy used by S1 is correct.

S1 also furthered the decapsulation process by reconstructing previous knowledge of Bayes' theorem with the concept of conditional probability. To begin, S1 determined the probability values of two possible events when solving Bayes' theorem. They were the event probability *of* $P(B \cap A^C)$ and $P(B^C \cap A^C)$. After writing the formula of Bayes' theorem, S1 did not use the values in the final calculation stage. S1 evaluated that the event possibilities of $P(B \cap A^C)$ and $P(B^C \cap A^C)$ did not belong to the total probability of event B. Figure 5 shows S1's de-encapsulation stage.

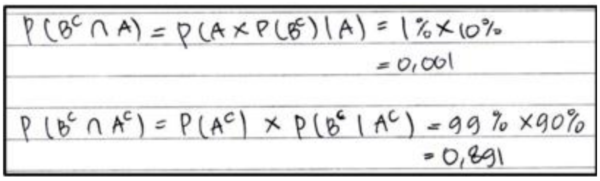


Figure 4. The De-encapsulation Results of S1 in Determining the Probability Values of Two Events

The interview results of S1 related to mental mechanisms in the de-encapsulation stage.

R: What did you think about calculating the total probability values of the positive mammogram test event?

S1: I calculated all probability of two intersecting events.

R: Then, did you use all probability values you calculated in determining the total probability?

S1: Hmmm... After checking the result, I found that only the probability of B with A intersection and the probability of B with complementary A intersection. Sorry madam, I wrote a lot.

S1's process in the problem-solving stage was correct. In this stage, S1 constructed the mental structure of the object. S1 promoted the encapsulation process by placing the process in the revealed objects in the written task and interview. S1 performed the encapsulation process by calculating the probability of a woman developing cancer if the mammography test results were positive. S1 used Bayes' theorem to solve the questions. Figure 4 shows S1's encapsulation results when determining the probability values using Bayes' theorem to determine the probability values.

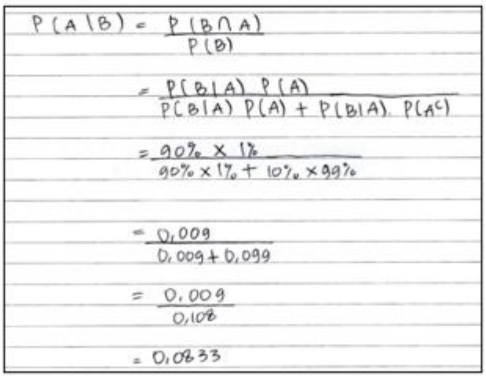


Figure 4. The Encapsulation Results of S1 When Determining the Probability Values with Bayes' Theorem.

The researchers interviewed the participant to determine S1's encapsulation process.

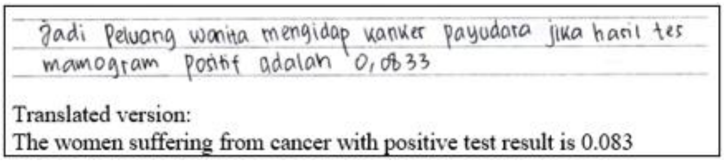
R: In your opinion, what is the probability of women who have cancer if the mammogram test results are positive? Tell me the process.

S1: I wrote the formula of Bayes' theorem then I substituted the values. After calculating, I obtained the probability value. It was 0.0833.

R: Does the result make sense?

S1: I do because the value, 0.0833, is between the estimated probability value.

In the confirmation stage of the response, S1 interpreted the calculation both orally and in writing. S1 was able to confirm the answers based on the written objectives accurately. Figure 5 shows S1's confirmation of the answer.


Jadi Peluang wanita mengidap kanker payudara jika hasil tes mamogram positif adalah 0,0833

Translated version:
The women suffering from cancer with positive test result is 0.083

Figure 5. The Results of the Phase of Confirmation of S1's Answer.

S1 reviewed the goal of the question again and the calculation result to conclude. S1 also believed in the conclusion drawn. The interview results with S1 about the stage of confirming the answer are presented below.

R: Have you confirmed your final answer?

S1: I have. I checked the questions and the objectives of the questions again. Then, I drew conclusions based on the data-processed and calculated results.

R: What conclusions did you draw?

S1: I concluded that the probability of women with cancer if the mammogram test results were positive was 0.0833.

R: Did you believe in your conclusion?

S1: I did because the result aligned with the question objective.

Figure 5 and the interview result show that S1 was able to draw a conclusion based on the results obtained. The participant also confirmed the calculation results based on the question objectives.

S1 was able to promote a thematization of mental mechanisms on the mental schema structure. The evidence for this process was S1's ability to identify some events, create personal representations, calculate event probability, and calculate conditional probability. S1 applied conditional and total probability concepts to determine the probability values using Bayes' theorem accurately. These results showed that S1 promoted the correct problem-solving process. The results of S1's mental mechanism questioning can be found below.

R: After determining the probability values, what did you figure out of Bayes' theorem? Please tell me.

S1: The applied Bayes theorem was valid to calculate the probability of an event where some other events are still correlated and become the precondition of the current event.

The interview results show that S1 promotes thematization by linking the background knowledge and the reconstructed problem-solving process. Figure 6 is the result of the analysis of the problem-solving process with correct symbolic representation based on APOS theory.

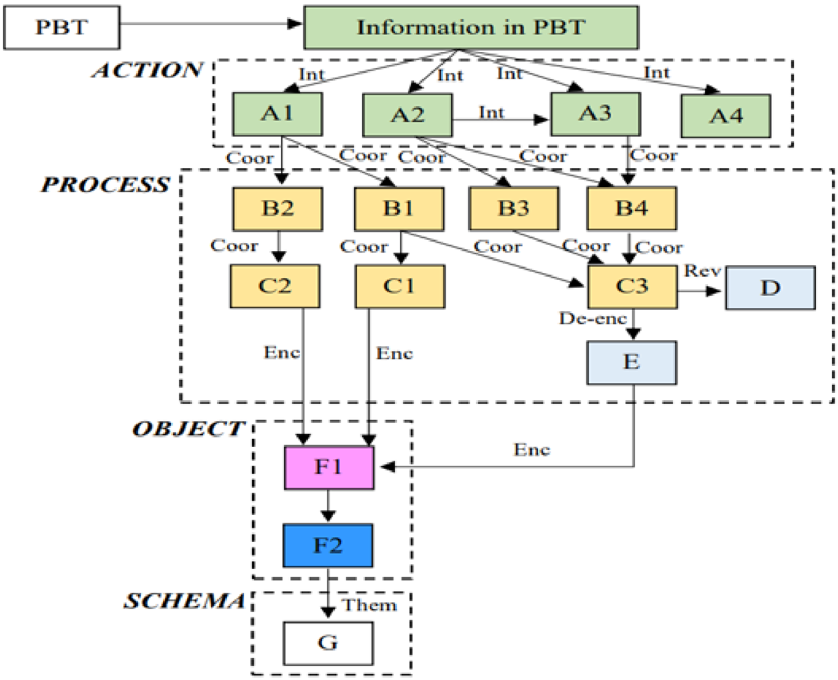


Figure 6. The Analysis of the Problem-Solving Process of S1

Table 2. The Remarks of Codes in Diagram 1

Graphic Codes	Remarks	Graphic Codes	Remarks
PBT	Problem-Solving Task	F1	Determining the probability value
A1	Identifying the probability of women suffering from cancer	F2	Interpreting and confirming the probability values
A2	Identifying the probability of women with positive mammogram test results if they were suffering from cancer	G	The applied Bayes' theorem was useful to calculate the probability of an event in which some other events are still correlated and become the requirement of the current event
A3	Identifying the probability of women with positive mammogram test results if they were not suffering from cancer	Int	Interiorization
A4	Identifying the objective: the probability of women suffering from cancer if the mammogram test results were positive	Coor	Coordination
B1	Creating an event analogy of women suffering from cancer	Rev	Reversal
B2	Creating an event analogy of women that did not suffer from cancer	De-enc	De-encapsulation
B3	Creating an analogy of a positive mammogram test result	Enc	Encapsulation
B4	Creating an analogy of a negative mammogram test result	Them	Thematization
C1	Correctly processing and presenting the events of women suffering from cancer	(green)	Reading and understanding the problem
C2	Processing and presenting the complementary events	(orange)	Representing the problem
C3	Processing and presenting the conditional events	(light blue)	Constructing problem-solving strategies
D	Recalling the conditional probability concept	(pink)	Solving the problem
E	Reconstructing the cognition of total probability and the intersecting event probability	(blue)	Confirming the answer

The Problem-Solving Process of S2 with Symbolic-visual Type Representation

The stage of S2's reading and understanding of the problem began with repeated reading activity. This S2's action mental structure occurred since S2 was confused to understand and determine the following step. From the interview results, S2 promoted the interiorization of mental mechanisms by correctly identifying the information and components of problems. S2's identification process showed the events and probability of women with cancer, women with positive mammogram test results, and women with false mammogram results. The interview results indicating the interiorization of S2 can be found below.

> *R: What action did you take when you received the questions?*
>
> *S2: I read it.*
>
> *R: How many times did you read the questions?*
>
> *S2: I read it many times because I was a bit confused about determining the following step.*
>
> *R: What information did you get from the questions?*
>
> *S2: People with breast cancer risk only have a 1% probability of developing cancer. Then, the positive test results showed that people who have cancer obtained a percentage of 90%. On the other hand, people with positive results but who did not have cancer obtained a percentage of 10%. The question was the probability of people who have breast cancer that developed cancer and obtained positive results.*

S2 could promote the second step in the problem-solving process. S2 created a mathematical model by analogy of A as the event of a positive mammography test result, event B as the event of women who have cancer, and event C as the event of women who do not have cancer. S2 created mathematical symbols and a tree diagram to clarify the situation of the question. At this stage, the observable mechanism was a coordinated mental mechanism toward the intelligible components of the question. Figure 7 shows S2's coordination results in the form of symbolic and visual representations.

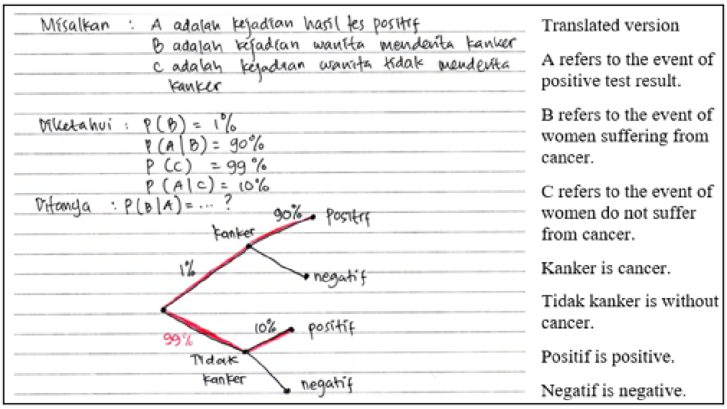


Figure 7. The Coordination Result of S2

The interview with S2 helped find out the mechanism of mental coordination. The interview results of S2 about the promoted coordination are presented below.

> *R: What did you think after obtaining the problem components?*
>
> *S2: I wrote the explanations about the content of the question.*
>
> *R: What was the next stage you did?*
>
> *S2: I created a mathematics model, symbol, and tree diagram.*
>
> *R: Why did you choose these representations to present problems?*
>
> *S2: I chose the tree diagram to facilitate me in explaining the inter-connected events with the probability.*

Figure 7 and the interview results show S2's mental structure process. In this case, the participant promoted coordinated mental development by creating a mathematical model of the events. S2 then presented the mathematical symbols and visual representations in a tree diagram. S2 used a visual representation to explain some events and ensure the probability value. This process shows that S2 was able to represent the problem with accurate symbolic and visual representations.

In the constructive problem-solving stage, S2 promoted the reversal of mental mechanisms. This mechanism was an attempt to remember the concept of total probability. S2 coordinated the concept with the internalized components of the problems. Figure 8 shows the results of S2's reversal in the form of a written test.

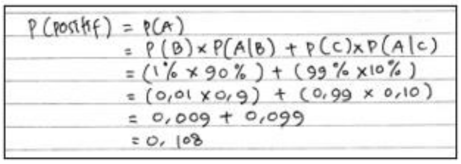


Figure 8. The Reversal Result of S2

The researchers interviewed S2 to uncover the mechanism of the advertised reversal of results. The results from the interview with S2 are given below.

R: What was your strategy to solve the problem?

S2: Firstly, I calculated the total probability values of the positive test event. I did this because the denominator of Bayes' theorem required total probability. After determining the total probability, I used Bayes' theorem to solve the questions.

The figure and the interview result show S2's mental process indicated by the reversal of mental mechanisms. S2 did this by recalling the concept of total probability. S2 calculated the total probability value by coordinating the internalized components of the problems. Then S2 used the results with the formula of Bayes' theorem. S2 had already done all stages of problem construction.

In the mental structure of the object, S2 promoted the problem-solving process. S2 promoted the mental encapsulation mechanism. S2 made the process a findable object in the written and oral responses. The results of S2's encapsulation took the form of written responses. S2 calculated the probability values using Bayes' theorem and the overall probability. Using the visual representation, S2 could accurately select the events from the situations in the question. Figure 9 shows S2's encapsulation result when solving the problem.

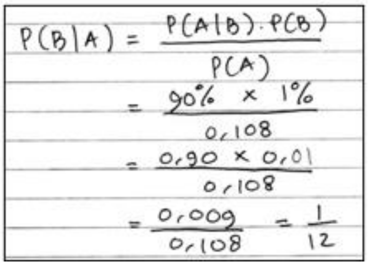


Figure 9. The Encapsulation Results of S2 in Determining the Probability Values

In addition to the written responses, the encapsulation step was also observed during the interview. The following quotations show the interview results showing S2's mental encapsulation.

R: In your opinion, what is the probability of women who have cancer if the mammogram test results are positive? Tell me the process.

S2: I wrote the formula of Bayes' theorem. Then, I substituted the probability values of the given elements of the question. I also used the results of the total probability calculation. After calculating the probability of a woman suffering from cancer if she has a positive mammogram test result, the obtained result is $\frac{1}{12}$.

R: Does the result make sense?

S2: It does because I did it based on the correct steps.

In the final problem-solving process, S2 promoted confirmation of the answer. S2 interpreted the calculation result in written and oral ways. S2 was able to confirm the answers based on the written objectives accurately. Figure 10 shows S2's confirmation of answers.

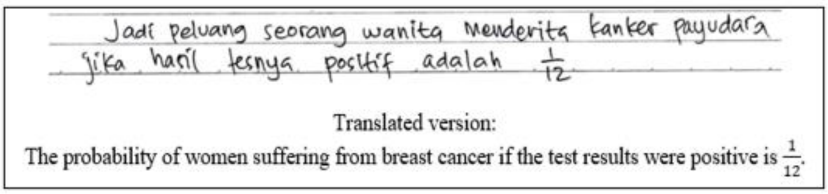


Figure 10. The Results of the Confirming the Answer Stage of S2

The interview results of S2's answer regarding the confirmation stage are as follows.

R: Have you confirmed your final answer?

S2: I have. I also wrote my conclusions after finding out the answers.

R: What conclusions did you draw?

S2: The probability of a woman who has breast cancer if the test is positive is $\frac{1}{12}$.

R: Did you believe in your conclusion?

S2: I did since I checked the process.

Figure 10 and the interview result show that S2 could infer accurately after determining the answer.

In the mental structure of the schema, S2 was able to promote the thematization of mental mechanisms. S2's ability to identify the components of the problems creates visual representations and calculates the probability values using Bayes' theorem. These results indicated that S2 promoted the correct problem-solving process. The interview results showed that S2 promoted theming by connecting the background knowledge and the reconstructed problem-solving process. The interview results with S1 about the thematization of mental mechanisms are as follows.

R: After determining the probability values, what did you figure out of Bayes' theorem? Please tell me.

S2: Dealing with the probability concept, if some events are parts of an event, then the applicable formula is Bayes' theorem.

Figure 11 shows the analysis of the problem-solving process with correct symbolic-visual representation based on the APOS theory.

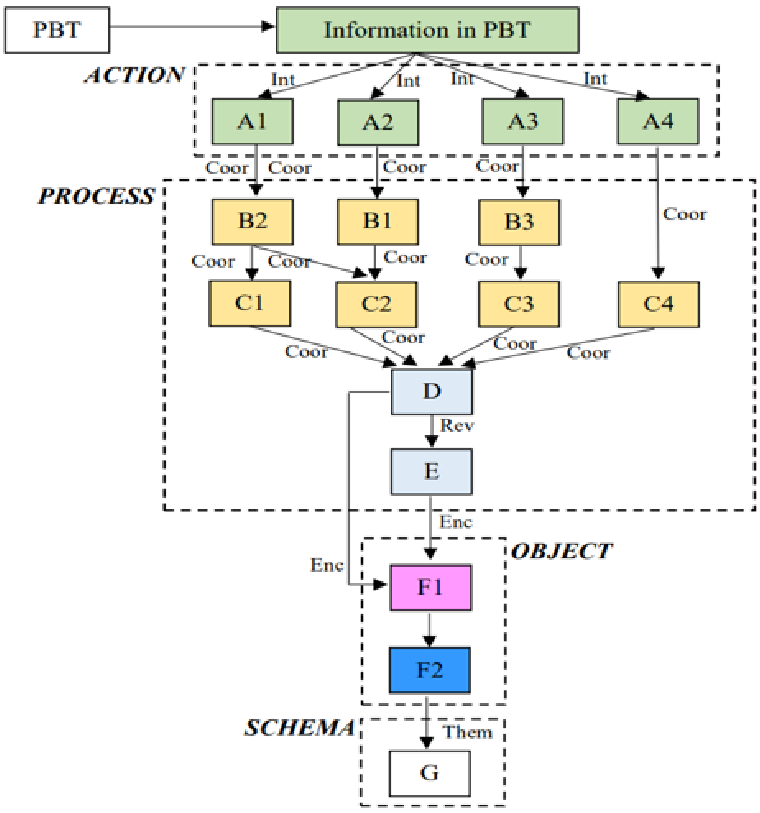


Figure 11. The Analysis of the Problem-Solving Process of S2

Table 3. The Remarks of Codes in Diagram 2

Graphic Codes	Remarks	Graphic Codes	Remarks
PBT	Problem-Solving Task	F1	Determining the probability with Bayes' theorem
A1	Identifying the probability of women suffering from cancer	F2	Interpreting and confirming the answers
A2	Identifying the probability of women with positive mammogram test results if they were suffering from cancer	G	Dealing with the probability concept, if some events are parts of an event, the applicable formula is Bayes' theorem
A3	Identifying the probability of women with positive mammogram test results if they were not suffering from cancer	Int	Interiorization
A4	Identifying the objective: the probability of women suffering from cancer if the mammogram test results were positive	Coor	Coordination
B1	Creating an analogy of women with positive test results	Rev	Reversal
B2	Creating an event analogy of women suffering from cancer	Enc	Encapsulation
B3	Creating an event analogy of women that did not suffer from cancer	Them	Thematization
C1	Correctly processing and presenting the events of women suffering from cancer		Reading and understanding the problem
C2	Processing and presenting the conditional events		Representing the problem
C3	Processing and representing the event probability of women that did not suffer from cancer		Constructing problem-solving strategies
C4	Processing and representing the event probability of women with positive results that did not suffer from cancer		Solving the problem
D	Creating visual representation		Confirming the answer
E	Recalling the formula of total probability		

Discussion

The first mental action structure in the problem-solving process was - reading and understanding the problem. The symbolic representation and the symbolic-visual representation read the question and wrote down the given information. The participants read the question repeatedly to understand the components of the problems and the intended goals, as shown by the results of previous studies, namely that excellent reading and reasoning skills are essential for students to solve written problems (Aljarrah, 2020; Pakarinen & Kikas, 2019).

The process is the second level of problem-solving construction. Students could write the goals of the questions and choose accurate representations to solve problems. The student's symbolic representation could promote the coordination of mental mechanisms by creating analogies and complementary events with some symbols. The student's symbolic representation uses symbolic representations when presenting problems. Previous studies have shown that symbolic representation is an essential problem-solving stage because representation can indicate a person's mathematical fluency (Cartwright, 2020; Ott et al., 2018). The results of symbolic representation in this study differ from those of Tobia et al. (2021), who found that symbolic intervention had an impact on matching students' mathematical ability with their peers but did not explain the role of symbolic representation in problem-solving.

Symbolic visuals student promoted coordination by creating event analogies with symbols, writing the probability values, and creating a tree diagram to visualize the situation in the questions. Students' symbolic-visual representation promoted representation by writing the symbols, such as variables, numbers, equations, and diagrams, to understand the situation in the question. The results of this study are consistent with those of previous studies, which found that visual representations can support mathematical skills in calculating and synthesizing mathematical sentences to solve problems (Boonen et al., 2014; Tam et al., 2019). Somewhat different from previous research that focused on mathematical reasoning, in which students first reasoned about the numerical data present in the problem, made symbolic representations, and visually represented the problem (Hoogland et al., 2018; Krawec, 2014).

In constructing problem-solving strategies, both students promoted the reversal of mental mechanisms. Both students recalled probability concepts, such as the concept of conditional probability. The conditional probability formula applied to determine the total probability in the last part was Bayes' theorem. The symbolic visual representation entered by the students also used the concept of total probability. The visual representation used a tree diagram to list all events based on the situation in the question. The symbolic representation typed by the students promoted the decapsulation process by reconstructing their knowledge of Bayes' theorem using the concept of conditional probability. Students rebuilt the complete knowledge to match it with the computational results.

Both participants promoted encapsulation of mental mechanisms by turning processes into an object. During encapsulation, students solved problems. Both participants could accurately perform mathematical calculations based on formulas, strategies, and chosen representations. Both participants could determine the final answers based on the goal of the question. Research focusing on computation skills in student-centered learning has shown a significant relationship with problem-solving skills (Pakarinen & Kikas, 2019). Counting skills help students' complete strategies and find final answers.

In the stage of confirming the answer, symbolic representation, and symbolic-visual representation typing, students were able to confirm the calculation results and draw conclusions based on the goal of the question. This process is supported by previous research that the final stage of problem-solving is to write the conclusion and communicate the results based on the interpreted data (Lee, 2017; Surya et al., 2017).

After the encapsulation process, the constructed objects must be connected to background knowledge to create a mental structure (E. D. Dubinsky & Mcdonald, 2001). In this study, two students, the students with symbolic representation and those with symbolic-visual representation, were able to promote the thematization of mental mechanisms in mental schema structure. Students with symbolic representation could thematize by generalizing the value of event probability. This event probability still had a connection, so it could be calculated using Bayes' theorem.

The symbolic-visual representation typed by the students promoted thematization. The clue was the capability to identify the components of the problems, create a visual representation, and calculate the probability value using Bayes' theorem. The results of this study follow previous research stating that schemas develop as connections between actions, processes, objects, and prior schemas (Arnon et al., 2014; Borji et al., 2018; Subanji & Nusantara, 2016).

The symbolic representation typed students and symbolic-visual representation typed students could complete all problem-solving processes. Both participants solved problems with processes influenced by cognitive style (Mora et al., 2021; Son & Fatimah, 2020). They performed the stages of understanding the problem, presenting it, constructing problem-solving strategies, solving the problem, and confirming the answer. The difference between the types was in the types of mathematical representation chosen. In symbolic representation, students could use mathematical symbols to represent problems. In the symbolic-visual representation of writing, students were able to represent the symbols and create diagrams to understand the situations of the question.

The participants were individuals with a reflective cognitive style. Thus, they tended to think before answering the questions. They did so patiently and carefully to obtain more information and apply a problem-solving strategy. They also liked to create problem analogies and needed more time to answer. Nevertheless, they were able to answer accurately. The results of this study are consistent with those of previous studies reporting that reflective individuals can carefully and holistically collect and analyze data (Mora et al., 2021), use a holistic analysis process (Warli, 2013), and include process information from tasks or problems by prioritizing a systematic strategy and solution (Margunayasa et al., 2019).

Conclusion

Based on the results and discussion, the reflective cognitive style students solve probability problems in five problem-solving stages. The stages were reading and understanding the problem, presenting the problem, developing problem-solving strategies, solving the problem, and confirming the answer. The new finding from this study was that the student's cognitive style influences the problem representation step in the problem-solving process. Based on the representation of the created problems, there are students with symbolic and visual representation styles. In the problem representation stage, students with symbolic representation use representations of mathematical symbols to support the problem-solving process. The indicators of students' symbolic representation were their ability to reconstruct the problem situation using symbols, numbers, and variables. In the problem representation stages, students' symbolic-visual representation was used as a symbolic representation. The indications were the use of symbols, notations, numbers, and visual representations in the form of a tree diagram. This study contributes to the students' problem-solving process and the use of different representations for each cognitive style.

Recommendations

Based on the obtained results from this research, the researchers recommend that the following steps be used when solving mathematical problems: reading and understanding the problem, presenting the problem, constructing problem-solving strategies, solving the problem, and confirming the answer. For further research, it is necessary to study the problem-solving process in other areas of mathematics, such as algebra and geometry, and to include the problem-solving process in different participants with different cognitive styles, namely impulsive, fast-accurate, and slow-accurate.

Limitations

This research only examines the problem-solving process with APOS theory on probability material. The participants of this research are limited only to mathematics students with a reflective cognitive style. Ideally, all students with impulsive, fast-accurate, and slow-inaccurate cognitive styles would also be studied.

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Authorship Contribution Statement

Rahayu: Conceptualization, design, data analysis, writing. Kartono: Editing/reviewing, supervision, final approval. Dwijanto: Critical revision of manuscript, supervision. Agoestanto: Data acquisition, reviewing, supervision.

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Appendix

Table A1. Point of Interview Question Problem-Solving Process Based on Action Process Object Schema Theory

Problem-solving Process	APOS Mental Structure	Mental Mechanism	Interview Points
Reading and understanding the problem	Action	Interiorization	1. What action did you take once you received the questions? 2. How many times did you read the questions? 3. What information did you get from the questions? Can you tell me?
Representing the problem	Process	Coordination	4. What is being questioned in the questions? 5. In your opinion, is the given information already enough to solve the problem? 6. What did you think after obtaining the problem components? 7. What was the next stage you did? 8. Why did you choose these representations to present problems?
Constructing problem-solving strategies	Process	Reversal	9. What was your strategy to solve the problem?
		De-encapsulation	10. What did you think about calculating the total probability values of the positive mammogram test event? 11. Then, did you use all probability values you calculated in determining the total probability?
Solving the problem	Object	Encapsulation	12. In your opinion, what is the probability of women who have cancer if the mammogram test results are positive? Tell me the process. 13. Does the result make sense?
Confirming the answer	Schema	Thematization	14. Have you confirmed your final answer? 15. What conclusions did you draw? 16. Did you believe in your conclusion? 17. After determining the probability values, what did you figure out of the Bayes' theorem? Please tell me.