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Preservice Teachers' Noticing Skills in Relation to Student Misconceptions in Algebra

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Abstract: Many students have misconceptions about mathematics, so preservice teachers should be developing the skills to notice mathematical misconceptions. This qualitative study analyzed preservice teachers' skills in noticing student misconceptions about algebra, according to three aspects of noticing found in the literature: attending, interpreting and responding. Participants in this study were seven preservice teachers from one university in the capital of Aceh province, Indonesia, who were in their eighth semester and had participated in teaching practicums. Data was collected through questionnaires and interviews, which were analyzed descriptively. The results revealed the preservice teachers had varying levels of skill for the three aspects of noticing. Overall, the seven preservice teachers' noticing skills were fair, but many needed further development of their skills in interpreting and responding in particular. This university's mathematics teacher education program should design appropriate assessment for preservice teachers' noticing skills, as well as design and implement learning activities targeted at the varying needs of individual preservice teachers regarding noticing student misconceptions, in order to improve their overall teaching skills.

Keywords: *Mathematical understanding, misconceptions, pedagogical content knowledge, preservice teachers, teacher education.*

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Introduction

Algebra is a topic taught in elementary school through to senior high school as a building block for various kinds of higher level mathematics (Choike, 2000). Students are required to understand algebra to increase their success in mathematics, as Booth et al. (2016) assert. Misconceptions about algebraic concepts pose a barrier to this understanding and therefore prevent them from proceeding to higher stages in mathematics. Several previous studies have shown that students hold misconceptions about algebra. Research by Bush and Karp (2013) found misconceptions in high school students of various countries regarding algebra. Many students misunderstand the equals sign (Byrd et al., 2015) and have problems with equality, inequality, negativity, variables, fractions, order of operations and functions (Booth et al., 2016). In 2020, Tendere and Mutambara noted that students simplified $2x^2 + 5x + 3$ to become $10x^2$, showing they mistakenly thought that $2x^2$ and $5x$ were like terms. Therefore, misreading and misunderstanding terms is also an issue.

Besides high school students, preservice teachers also experience barriers in learning algebra. Many preservice teachers have an insufficient understanding of algebra (Brown & Bergman, 2013; Tanisli & Kose, 2013); experience difficulties in choosing word problems suitable for linear graph structures (Kar, 2016); and struggle to identify linear equations as well as answer questions related to the rate of change in the context of algebra (Stump, 1996, as cited in Zuya, 2017). In contrast, the Association of Mathematics Teacher Educators (AMTE, 2017) gives standards for beginner mathematics teachers, stating that they should have a strong knowledge of concepts, pedagogy, as well as a good understanding of their students' mathematical knowledge. The National Council of Teachers of Mathematics (NCTM, n.d.) also argues that effective mathematics teachers should use evidence of student thinking to assess the progress of their mathematical understanding and adjust instruction to support learning. There is therefore a need for preservice teachers to develop their own understanding of algebra to be able to build correct student understanding and prevent misconceptions.

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One of many essential skills for teachers according to Thomas et al. (2017) is professional noticing. Noticing skills are defined as skills for identifying and providing feedback on student work during the teaching and learning process. Miller (as cited in Amador, 2014) states that noticing is the ability of teachers to understand each student's thinking processes to ascertain their level of understanding, including the difficulties and misconceptions that the student experiences during learning. Jacobs et al. (2010) proposed three interrelated skills that comprise the noticing process: paying attention, interpreting and responding. McDuffie et al. (2018) also refer to what teachers see (attending), how they understand student thinking and ideas (interpreting), and the teacher's decision to act (responding). These interconnected skills are put forward as essential for both preservice and inservice teachers in order to be successful educators.

Various factors influence a teacher's noticing skills, including knowledge, experience and cultural background (Ding & Domínguez, 2016; Jacobs et al., 2010; Louie, 2018). Knowledge here refers to knowledge about content, pedagogy as well as pedagogical content knowledge (PCK) (Newsome et al., 2017; An & Wu, 2012). PCK is closely related to noticing (Ding & Domínguez, 2016; Kilic, 2018). A teachers' experience of learning at university also influences their noticing skills. Hiebert et al. (2007) explain that experiences gained as a university student influenced how preservice teachers interpreted their students' understanding, a key aspect of noticing. Jacobs et al. (2010) examined the noticing skills of in-service and preservice teachers during video learning activities as well as their responses to students' procedures in solving whole-number operation problems. They concluded that noticing skills were influenced by both teaching experience and professional development activities.

Wessels (2018) and Barnhart and van Es (2015) comment on a lack of noticing skills and analysis of teaching and learning activities by preservice teachers in South Africa and the United States. While experienced teachers can better observe, understand and use analyses of important events in complex situations to adjust their teaching (Barnhart & van Es, 2015), preservice teachers tend to have difficulty in attending to and interpreting student understanding (Star & Strickland, 2008). Preservice teachers focus more on learning as a whole, rather than understanding, interpreting, and responding to student ideas in order to achieve learning goals (Erikson, as cited in Barnhart & van Es, 2015). However, when receiving guidance, preservice teachers are able to analyze and understand student ideas and thought processes at a level on par with inservice teachers (Barnhart & van Es, 2015; Levin et al., 2009). This mentoring in developing noticing skills can be provided during teacher preparation programs (Jacobs et al., 2010; Stockero et al., 2020).

In order to determine whether such guidance or any other intervention is needed in the province of Aceh, Indonesia, studies must first be undertaken to measure current preservice teachers' noticing skills. This study was therefore designed to measure the noticing skills of preservice teachers studying at a state university in the provincial capital Banda Aceh, Indonesia, to provide a baseline for potential improvement of the university's current teacher education curriculum. This research is unique in that it is the first in Aceh province to measure and assess preservice teacher noticing skills in a university's teacher education program. In the light of the points above, the writers aimed to achieve the following research goal: Describe and interpret the level of noticing skills of preservice teachers when analyzing students' algebraic misconceptions.

Literature Review

Algebraic Misconceptions

Previous research describes school students making mistakes and experiencing common misconceptions regarding algebraic material (Bagni, 2001; Booth et al., 2016; Sahin & Soylu, 2011). Misconceptions may begin as misunderstandings or overgeneralizations, which become systematized (Leinhardt et al., 1990) and result in wrong knowledge constructs (Suparno, 1997). Poor teaching, student forgetfulness and informal thinking can also lead to misconceptions (Allen & Goldsby, 2008). Pines (1985, as cited in Confrey, 1990, p. 110) explains that misconceptions are "contingent upon a certain existing conceptual framework" which does not align with that considered appropriate by experts in the field. Nesher (1987) holds that students' misconceptions, flagged by their errors, should not be confronted all at once in order to avoid confusion. Instead, the teacher should link new knowledge to the accurate parts of the student's existent conceptual framework. Several elements of algebra which are commonly misunderstood or misapplied have been mentioned in the introduction to this paper. Egodawatte (2011) suggests that there are four aspects of algebra where misconceptions are most likely to occur, specifically, algebraic expression, use of variables, algebraic equations and word problems.

One of the misconceptions regarding algebraic expressions is that students often confuse algebraic addition with algebraic multiplication. For example, some students try to simplify algebraic expressions like $4 + 3x^2$ as $7x^2$ (Allen & Goldsby, 2008). This error occurs because students mistakenly consider the original expression to not be in its simplest form. In addition, Herutomo and Saputro (2014) show that students wrongly believe that simplifying algebraic expressions utilizes the same process as simplifying arithmetic problems whose final result is a single-digit number. Another misconception that students often possess is in regards to distributive rules. Students often mistakenly simplify the form $x^2 - 2(x - 3)$ to $x^2 - 2x - 6$ (Allen & Goldsby, 2008), and the form $3(2x - 4)$ to become $6x + 12$ instead of $6x -$

12 (Norton & Irvin, 2007). This error in applying distributive rules develops as a result of weak understanding of integer operations. Misconceptions also occur with the addition or subtraction of algebraic fractions of different denominators (Allen & Goldsby, 2008). Students in this case mistakenly believe that rules for multiplying fractions can be applied to addition problems, forgetting that in adding fractions, they must first convert the fractions with unlike denominators. Two types of misconceptions were examined in this study: those involving algebraic expressions and those involving variables, as presented in Table 1.

Pedagogical Content Knowledge (PCK)

Teachers need to have thorough knowledge of the content to be taught; the characteristics of their students as learners; as well as pedagogical knowledge (NCTM, n.d.; Kilic, 2018). Shulman (1987) terms this knowledge as pedagogical content knowledge (PCK), defining PCK as the intersection of a knowledge base regarding content and teaching styles with pedagogical reasoning and action, which includes knowledge of student thinking, including any misconceptions. Lack of teacher PCK has a detrimental impact on student understanding and overall learning. PCK specifically requires an understanding of student conceptual frameworks as influenced by previous learning, which is useful in analyzing student misconceptions. PCK leads to good noticing skills because it directly impacts the way teachers make choices while teaching. Teachers should be aware of what is happening in class, paying attention to opportunities to support student learning by reviewing content, handling difficulties, overcoming misunderstandings, and developing student thinking through questions (Kilic, 2018), four skills which center around the teacher's PCK.

Preservice teachers also need to develop and make use of content knowledge, pedagogical knowledge, as well as the result of their convergence: pedagogical content knowledge (PCK) (Sujadi et al., 2019). Schäfer and Seidel (2015) explain that preservice teachers need to connect student learning events with pedagogical ideas, with the help of professional knowledge. Besides training in pedagogy and content, preservice teachers must be committed to increasing their knowledge about students (Gichobi & Andreotti, 2019), an aforementioned aspect of PCK. Since beginner mathematics teachers must be already able to use PCK regarding student understanding (AMTE, 2017), mathematics teacher education programs must also ensure that their graduates possess sufficient PCK to be effective beginner mathematics teachers. Kleickmann et al. (2013) deem it necessary for teacher education programs to provide opportunities for preservice teachers to build their PCK, which is the basis for preservice teacher noticing skills.

Noticing

Jacobs et al. (2010) define noticing as the skills involved in attending to, interpreting, and responding to student understanding. This is supported by Callejo and Zapatera (2016), who hold that mathematics teachers' professional noticing is the ability to understand and analyze students' mathematical reasoning involving the reconstruction of the steps taken to solve a problem and an inference of their understanding from what they wrote, said, or did. The first skill, attending, is related to the ability to observe or identify essential learning events. In other words, teachers should "understand the students' understanding" (Philipp, 2014, p. 285). This skill requires mathematics teachers to pay attention to an important part of learning: students' mathematical strategies, including the mathematical concepts they make use of. Understanding the strategies used by a student enables their teacher to ascertain their ideas or thought processes in solving problems (Fernández et al., 2013). Therefore, this aspect of noticing centers around teacher understanding of student thinking.

The subsequent skill, interpreting, requires teachers to make inferences from their understanding of student thinking. Interpreting means that teachers understand more deeply what happened: for example, what students did or did not understand. In addition, teachers analyze why students experience difficulties with a concept (Fernández et al., 2013). In forming their interpretations, teachers need to first understand student strategies, then have sufficient knowledge and skills in mathematics to explain student understanding of mathematical concepts (Jacobs et al., 2010). In other words, pedagogical content knowledge (PCK) is needed to interpret student understanding (Ding & Domínguez, 2016). The interpreting aspect of noticing therefore focuses on analyzing, deeply understanding and uncovering reasons for student thinking.

The last skill involved in noticing is responding, or providing appropriate assistance so that student understanding of mathematical concepts improves (Fernández et al., 2013). Teachers design learning that helps students amend their misunderstandings and construct acceptable understanding (Levin et al., 2009). Assistance is defined in this study as actions to reduce or eliminate misconceptions experienced by students, especially those concerning algebra. There are many types of assistance teachers can provide to help students overcome misconceptions: one recommended by Baser (2006) and Swedosh and Clark (1997) being the cognitive conflict approach. This approach requires teachers to take three steps: raise awareness of student misconceptions by providing opportunities for students to test their existing understanding; give space for them to re-consider their thinking to notice where it is not correct; and guide students to make changes to their conceptions. Responding is a necessary part of teachers' adjusting learning to suit their students' individual needs. Crespo (2000) explains that teachers who do not listen to or understand student thinking processes tend to carry out previously planned teaching and learning activities without regard to students' current thought processes. The result is that students learn with less understanding because teachers are not connecting new material with the understanding students already possess. The final aspect of noticing therefore involves designing effective future learning based on teachers' interpretations of students' present understanding, to help them overcome misconceptions.

Several studies conclude that noticing skills should be focused on specifically and deliberately by preservice teachers so that they can better help students learn. These skills require PCK as a foundation but go beyond mere knowledge into the realm of how this knowledge of the student's thinking is used by the preservice teacher to teach new material. Barnhart and van Es (2015) hold that preservice teachers should understand and apply the whole process of identifying, analyzing and responding to student understanding, because simply attending to student misconceptions is not enough to overcome erroneous student thinking: analyzing and responding to their thinking are also key components of teaching. Kilic (2018) investigated the noticing skills of preservice teachers and found that they tended to only pay attention to student mistakes and strategies. Similarly, Kılıç (2019) found that preservice teachers scored higher on attending to student mistakes than on interpreting these mistakes or giving helpful responses. Kılıç further asserts that most preservice teachers studied had not mastered all three aspects of noticing, with most only focusing on their pedagogical skills and the interactions occurring in class, without seeking student opinions. The authors conclude that noticing skills should be a key focus area for preservice teacher training, especially the second two aspects involving analysis and appropriate responses. The weakness of these prior studies on noticing skills in preservice teachers is that they were centered around locations such as USA, Europe, Turkey, Australia and Africa, with no prior research being conducted in Indonesia. The authors chose to focus on preservice teachers' noticing skills in Indonesia in regards to analyzing misconceptions experienced by students regarding algebraic expressions.

Methodology

Research Design

This descriptive qualitative research aims to describe phenomena experienced by participants in depth (Creswell & Creswell, 2018). The researchers describe the participants' ability to notice students' answers in algebra according to the three aspects of noticing (attending, interpreting, and responding). The researchers used long-answer questionnaires and also conducted interviews with each participant to obtain comprehensive data.

Participants

Participants in this study were seven preservice teachers out of a total of 90 in their cohort. The participants consisted of five women and two men, aged 21-22 years. They all had a Grade Point Average (GPA) of 3.00 or more and were willing to volunteer as participants. According to Johar et al. (2017), seventh-semester students in this university's education program have already completed four months of teaching practicums. The seven participants had finished their final practicums as they were all in their eighth semester. Therefore, they were considered to have sufficient knowledge and experience in teaching to be able to analyze student answers. The participants have been coded as Participant A (PA), Participant B (PB), Participant C (PC), Participant D (PD), Participant E (PE), Participant F (PF), and Participant G (PG).

Instruments

Most studies on the noticing skills of preservice teachers either utilized or commented on video as an educational tool (Ding & Domínguez, 2016; McDuffie et al., 2014; Star & Strickland, 2008; Taylan, 2015); made use of long answer questionnaires (An & Wu, 2012; Brown & Bergman, 2013; Gichobi & Andreotti, 2019; Matamoros et al., 2015), or combined the two above methods (Jacobs et al., 2010). This study utilized a response sheet with long answer questions and interviews of the participants. The response sheet contained nine stimuli which were in fact local high school student's wrong answers to three different algebraic problems. The questions posed by the response sheet regarding each erroneous student answer were:

1. Describe in detail what the student did to solve the problem given,
2. Please describe the ability and understanding of the student based on their response,
3. If you were this student's teacher, explain how you would respond to this student and how you would plan a lesson to teach algebraic concepts.

The stimuli had been created in the following manner. Several algebraic problems had been synthesized from those of Allen and Goldsby, (2008), Egodawatte (2011), Schnepper and McCoy (2014), Zuya (2014) and Herutomo and Saputro (2014), then given to 36 junior high school students in Aceh, Indonesia. The students' answers then were analyzed by the researchers, and the three algebraic problems having the most errors in answers were identified. From each of these problems, three student answers were selected to become stimuli in the instrument because they represented separate types of student misconceptions. These misconceptions related to: associative properties of algebraic expressions, i.e. simplifying algebraic expressions; multiplication involving algebraic fractions; and the addition of algebraic fractions. The variations of wrong student answers to the three problems selected are given in Table 1.

Table 1. Stimuli: Variations of Erroneous Student Answers

No.	Problem	Variations of student answers
1.	$(2x - y) + y$	Answer I: $(2x - y) + y = 2x(y) - y(y) = 2xy - y^2$
		Answer II: $(2x - y) + y = 2x - (y + y) = 2x - 2y$
		Answer III: $(2x - y) + y = 2xy^2$
2.	$A\left(\frac{1}{A}\right)$	Answer I: $A\left(\frac{1}{A}\right) = A \div \frac{1}{A} = \cancel{A} \times \frac{\cancel{A}}{1} = 1$
		Answer II: $A\left(\frac{1}{A}\right) = A^2 + 1$
		Answer III: $A\left(\frac{1}{A}\right) = \frac{A}{A^2}$
3.	$\frac{A}{B} + \frac{A}{C}$	Answer I: $\frac{A}{B} + \frac{A}{C} = \frac{A+A}{B+C} = \frac{A^2}{BC}$
		Answer II: $\frac{A}{B} + \frac{A}{C} = \frac{2A}{BC}$
		Answer III: $\frac{A}{B} + \frac{A}{C} = AC + AB = 2A + BC$

Data Collection

Data was obtained by giving the seven participants an instrument in the form of a response sheet with three algebraic problems, each with three variations of erroneous high school student answers (see Table 1, above). Participants were required to analyze each of the nine student answers according to long answer questions that focused on the three aspects of noticing (Jacobs et al., 2010). After the participants completed the response sheet, they were interviewed twice each to give them a chance to explain more in-depth the analysis which they had provided in writing.

Data Analysis

The written responses and interview responses of participants were assessed using the rubric in Table 2, which has been adapted from Jacobs et al. (2010).

Table 2. Preservice Teachers' Noticing Skills Rubric

Aspect	Level	Description
Attending	Evidence (1)	The response describes most of the mathematically important details of the student's steps or strategies in solving the problem.
	Lack of evidence (0)	The response does not describe details of how the student solved the problem, omitting the student's steps or strategies.
Interpreting	Robust evidence (2)	The response explains the student's understanding in depth or from various points of view.
	Limited evidence (1)	The response explains the student's understanding but is too broad or gives overgeneralized comments on the student's answer.
	Lack of evidence (0)	The response does not explain the student's understanding.
Responding	Robust evidence (2)	The plan assists the student to build correct understanding, including specific explanations of how their misconceptions could be explained, tested or changed.
	Limited evidence (1)	The plan contains general assistance to improve student understanding, without specific explanations linking to the student's misconceptions.
	Lack of evidence (0)	The plan does not relate to the student's understanding or misconceptions.

The data analyzed was a combination of written and oral responses, with both types of data being given consideration in order to gain as deep an understanding of the preservice teacher's skills in noticing as possible.

Results

The participating preservice teachers were asked to write about high school students' answers to three problems about algebraic expressions. For each problem, three types of student errors were provided, as shown in Table 1. The question prompts for preservice teachers' written responses were aimed at revealing their attending, interpreting and responding skills.

Attending

Preservice teachers wrote responses to Question 1: "Describe in detail what you think about the student's answer" in relation to the 9 erroneous student answers. Using the rubric in Table 2, the preservice teachers' scores for attending to each student's understanding were determined, with the results presented in Table 3 below.

Table 3. Participants' Scores for Attending Skills

Participant	Responses to student answers for problem 1			Responses to student answers for problem 2			Responses to student answers for problem 3			Median	Overall Level
	I	II	III	I	II	III	I	II	III		
PA	1	1	1	1	1	1	1	1	1	1	Evidence
PB	1	1	1	1	1	1	1	1	1	1	Evidence
PC	1	1	1	1	1	1	1	1	1	1	Evidence
PD	1	1	1	1*	1	1	1	1	1	1	Evidence
PE	1	1	1	1	1	1	1	1	1	1	Evidence
PF	1	1	1	1	1	1	1	1	1	1	Evidence
PG	1	1	1	1	1	1	1	1	1	1	Evidence

Table 3 shows that there was evidence that all participants attended to the students' thinking. The scores were determined by the researchers according to the rubric in Table 2 which has only two categories for attending based on Jacobs et al., (2010). All seven of the preservice teachers explained the mathematically important details of each student's steps or strategies in solving each problem. A sample written response and interview explanation has been selected and will be discussed below. It is marked in the table with and asterisk (*) and involves the student answer I for problem 2, as shown in Figure 1, below:

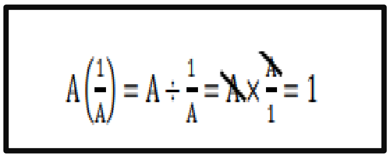
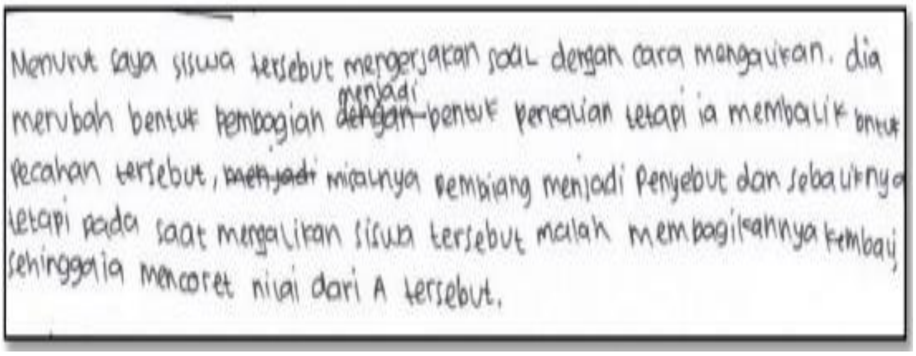


Figure 1. Problem 2, Student Answer I

Preservice teacher PD wrote their observation regarding the stimulus in Figure 1: this observation can be viewed in Figure 2, below.



Translation:

I think the student is working on the problem by multiplying. They changed the division to multiplication, but they reversed the fraction, for example the numerator became the denominator and vice versa, but when multiplying the student instead divided it again so he crossed out the A.

Figure 2. PD's Response to Question 1: "Describe in detail what the student did to solve the problem given" for Problem 2, Student Answer I

PD explained their answer during an interview as shown in the following excerpt:

Q : Please explain your written response to the student's answer.

PD : The result is correct. But the student's mistake is that the student thinks this parenthesis is a division. They wrote A divided by $\frac{1}{A}$. It is actually multiplication, A times by $\frac{1}{A}$. So, because they think this is division, they wrote A divided by $\frac{1}{A}$. Their concept is correct; for example, if you want to change the division into times, then reverse it, the denominator becomes the numerator, and the numerator becomes the denominator.

In the interview, PD explained their response in detail to the student's answer in Figure 1, including a description of the steps the student had taken and their thinking processes involving multiplying algebraic fractions, including the misconception that a parenthesis means to divide. Therefore, PD satisfied the descriptor for the "evidence" level for attending. PD did the same for each of the other student responses, as did the other six preservice teachers.

Interpreting

The responses of the participating preservice teachers in regards to the second question, testing their interpreting skills, were also analyzed by the rubric in Table 2 which follows the categories of Jacobs et al. (2010): "robust evidence" (2), "limited evidence" (1), and "lack of evidence" (0). Examples have been presented from one preservice teacher from each category for problem 3, student answer I, even though the overall levels of all participants were only spread across two categories: robust and limited evidence. The samples have been marked with asterisks as follows: * for robust evidence, ** for limited evidence, and *** for lack of evidence, as seen in Table 5.

Table 5. Participants' Scores for Interpreting Skills

Participants	Student answers to problem 1			Student answers to problem 2			Student answers to problem 3			Median	Overall Level
	I	II	III	I	II	III	I	II	III		
PA	2	2	2	2	2	2	2*	2	1	2	Robust evidence
PB	1	2	2	2	1	2	1**	1	1	1	Limited evidence
PC	2	2	1	2	2	2	1	1	1	2	Robust evidence
PD	2	2	2	2	2	1	1	1	1	2	Robust evidence
PE	2	2	1	2	1	1	2	1	1	1	Limited evidence
PF	2	2	2	2	1	1	0	0	1	1	Limited evidence
PG	2	2	1	2	1	0	0***	0	0	1	Limited evidence

According to Table 5, although all preservice teachers could show robust evidence for interpreting student understanding some of the time, only PA, PC and PD were able to give robust evidence for the majority of student answers. They also each had a median of 2, so were evaluated as having robust evidence overall. PB and PE showed both robust and limited evidence, but each had a median of 1 and a majority of limited and not robust evidence, so their overall level was easy to determine. However, PF and PG's responses were more varied, having scores in all three categories. PF's median score was 1, and they only had robust evidence four out of nine times, so the researchers found the best fit for their overall level was found in the limited evidence descriptor. PG had even less of the robust and more lack of evidence of interpreting than PF, but since their median was still 1 and it cannot be said that they were completely lacking in evidence due to the fact that they only scored 0 four of nine times, the best fit for PG overall was also the limited evidence category. It can be concluded that the majority of the preservice teachers were only able to give limited evidence of interpreting skills. The samples given below are from PA as the participant with the most robust evidence; from PB, representing the higher end of the limited level; and from PG, representing the lower end of the same. Their written analyses are given in Table 6, below.

Table 6. Preservice Teachers' Responses to Question 2: Interpreting

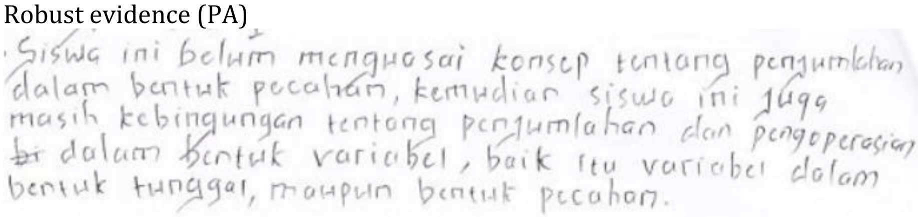
Student answer	Preservice teachers' responses to: "Please describe the ability and understanding of the student based on their response."
$\frac{A}{B}+\frac{A}{C}=\frac{A+A}{B+C}=\frac{A^2}{BC}$	Robust evidence (PA)  Siswa ini belum menguasai konsep tentang penjumlahan dalam bentuk pecahan, kemudian siswa ini juga masih kebingungan tentang penjumlahan dan pengoperasian ~~bi~~ di dalam bentuk variabel, baik itu variabel dalam bentuk tunggal, maupun bentuk pecahan. Translation: This student has not mastered the concept of addition in the form of fractions, and this student is also still confused about addition and operations in the form of variables, both variables of singular [non-fractional] and fractional forms.

Table 6. Continued

Student answer	Preservice teachers' responses to: "Please describe the ability and understanding of the student based on their response."
$\frac{A}{B}+\frac{A}{C}=\frac{A+A}{B+C}=\frac{A^2}{BC}$	Limited evidence (PB) Menurut saya, siswa memiliki pemahaman yang salah mengenai penjumlahan pada pecahan Translation: In my opinion, the student has an incorrect understanding of addition of fractions.
$\frac{A}{B}+\frac{A}{C}=\frac{A+A}{B+C}=\frac{A^2}{BC}$	Lack of evidence (PG) Siswa sudah mempunyai kemampuan yang cukup baik meskipun masih salah dalam menyelesaikan soalnya. Siswa sudah mempunyai kemampuan dasar meskipun salah menggunakannya. Translation: The student has a fairly good ability even though they are still incorrect in solving the problem. The student has the basic skills even though they use them incorrectly.

Table 6 shows that PA could analyze the student's answers to question 1 from various points of view. PA understands the reasons behind the student's answer, including their mistakes. It can be concluded that PA explains the student's understanding in depth because they refer to the concepts they are struggling with, such as algebraic processes and operations with fractions. On the other hand, participant PB's analysis shows limited evidence of interpreting as it lacked detail. PB's analysis of student answers was too short and general since they only mentioned operations on fractions. For this particular sample, preservice teacher PG lacked evidence of interpreting skills because their analysis did not explain the student's answer other than saying it was incorrect. PG also wrote that the student had good basic knowledge, when in fact, the student understood neither the necessary algebraic processes nor fraction operations.

Participants PA, PB and PG also explained their understanding of this student problem in their interviews. The following excerpt from the translation of PA's transcript gives their elaboration.

Q : What do you think about this student's understanding of solving algebraic problems?

PA : This student still does not understand the concept of addition in the form of fractions. This student is still confused about addition and subtraction in the form of non-fractional and fractional variables.

Q : What do you mean, variables in non-fractional form or fractional form?

PA : I mean, a variable in a non-fractional form is without fractions, such as $\mathrm{A}+\mathrm{A}$ and $\mathrm{A}+\mathrm{B}$, while in fraction form, it is like $\frac{\mathrm{A}}{\mathrm{B}}+\frac{\mathrm{B}}{\mathrm{C}}$. In the students' opinion, $\mathrm{A}+\mathrm{A}=\mathrm{A}^2$, it should be 2A.

Q : What do you mean, being confused with fractional forms?

PA : The student made a mistake when finding the result of $\frac{\mathrm{A}}{\mathrm{B}}+\frac{\mathrm{A}}{\mathrm{C}}$ which should be $\frac{\mathrm{AC}}{\mathrm{BC}}+\frac{\mathrm{AB}}{\mathrm{BC}}$.

In comparison to PA, in their interview, PG stated that "the student gave the final denominator as BC, which means they have basic abilities". But in fact, the student wrote $\mathrm{B}+\mathrm{C}=\mathrm{BC}$, and PG was unable to give details about this error, much less their misconceptions or the reasons for them. Thus, the interpretation of PG for this student answer to problem 3 is at the lowest level, even though PG was deemed at the limited level overall.

Responding

The data related to the preservice teachers' plans to respond to students with assistance were also classified as having robust, limited, or a lack of evidence for this skill. For each level, one preservice teacher's responses were selected for inclusion in this paper. The selected participants are marked with * for robust evidence, ** for limited evidence, and *** for lack of evidence in Table 7.

Table 7. Participants' Scores for Responding Skills

Participants	Responses to student answers for problem 1			Responses to student answers for problem 2			Responses to student answers for problem 3			Median	Overall Level
	I	II	III	I	II	III	I	II	III		
PA	2	2	1	2	2	2	2*	2	1	2	Robust evidence
PB	1	1	1	1	1	1	1**	1	1	1	Limited evidence
PC	1	1	1	1	1	1	1	1	1	1	Limited evidence
PD	1	1	1	1	1	1	1	1	1	1	Limited evidence
PE	2	1	1	2	1	1	1	1	1	1	Limited evidence
PF	2	2	2	1	1	1	1	1	1	1	Limited evidence
PG	1	1	1	1	0	0	0***	0	0	0	Lack of evidence

Table 7 reveals that only one preservice teacher gave robust evidence for their responding skills: PA, while another one preservice teacher was overall deemed lacking evidence. The remaining preservice teachers' responses were evaluated by the researchers as having limited evidence. In Table 8, the written responses of PA, representing the robust level; of PB, representing the limited level; and of PG, representing the level of lack of evidence are displayed.

Table 8. Preservice Teachers' Responses to Question 3: Responding

Student answer	Preservice teacher responses to: "If you were this student's teacher, explain how you would respond to this student and how you would plan a lesson to teach them algebraic concepts."
$\frac{A}{B}+\frac{A}{C}=\frac{A+A}{B+C}=\frac{A^2}{BC}$	Robust evidence (PA) Untuk siswa ini, mungkin terlebih dahulu saya akan menguatkan pemahaman mereka tentang operasi dalam bentuk pecahan, contoh : $\frac{1}{2}+\frac{1}{2}\neq\frac{1+1}{2+2}$ dan $\frac{2}{3}+\frac{1}{2}\neq\frac{2+1}{3+2}\neq\frac{2\times 1}{3\times 2}$ Translation: For this student, perhaps I would first strengthen their understanding of operations in fractions, for example: $\frac{1}{2}+\frac{1}{2}\neq\frac{1+1}{2+2}$ and $\frac{2}{3}+\frac{1}{2}\neq\frac{2+1}{3+2}\neq\frac{2\times1}{3\times2}$.
$\frac{A}{B}+\frac{A}{C}=\frac{A+A}{B+C}=\frac{A^2}{BC}$	Limited evidence (PB) Saya akan menjelaskan kembali cara penjumlahan pecahan. dimulai dari yang paling sederhana. yaitu memiliki penyebut yang sama hingga yang berbeda penyebut. Translation: I will explain again how to add fractions, starting from the simplest, which is with the same denominator, to ones with different denominators.
$\frac{A}{B}+\frac{A}{C}=\frac{A+A}{B+C}=\frac{A^2}{BC}$	Lack of evidence (PG) Saya akan melatih lagi atau mengajarkan siswa dengan strategi- strategi dengan model pembelajaran yang ampuh Translation: I will drill again or teach the student again using strategies, with powerful learning models.

The following is an excerpt from the translation of PA's interview transcript.

Q : If you were the teacher of these students, explain what would you do with this student?

PA : For this student, I would strengthen their understanding of operations with fractions. For example: $\frac{1}{2}+\frac{1}{2}\neq\frac{1+1}{2+2}$ and $\frac{2}{3}+\frac{1}{2}\neq\frac{2+1}{3+2}\neq\frac{2\times1}{3\times2}$.

Q : What do you mean by "strengthen"?

PA : I would help the student understand better by giving examples using numbers so that the student realizes that the results of the process do not make sense. After the analogy, I will ask the student to find a way of adding fractions involving variables.

PA's written response and oral elaboration gave a plan including specific explanations aimed at helping the student better understand operations with fractions involving variables. Their explanation included a test or disproof of the student's misconception regarding adding fractions, which is an example of cognitive conflict. PA's responses to this student's answer fit nicely in the highest level of the rubric in Table 2, as they gave robust evidence for responding skills. Seven out of nine times, PA's answers showed similarly robust evidence. Therefore, the best fit overall for PA is "robust evidence". PB, on the other hand, only gave a broad brush stroke plan to explain addition of fractions in general and did not link their plan to the student's thinking process or their misconceptions. In their interview, PB added, "I would give an example of how to add fractions, a simple one, that is with the same denominator, then I would also give an explanation about the addition of fractions with different denominators. Then I'd drill my student some more". PBs' responses showed an awareness of the algebraic concept the student needed to grasp, but their plan did not specifically mention how to relate the coaching material with the student's thinking process or misconceptions, so there is only limited evidence of the preservice teacher being able to respond. As for PG, some of the time, they gave limited evidence, but for the majority of the student answers, evidence of PG's responding skills responding was lacking. PG's response in Table 8 did not give a clear plan nor a solution or response to the student's misconception. In an interview, PG reiterated, "I would use learning models like cooperative learning, problem-based learning, and the jigsaw strategy to make the student active". PG did not relate the above statement or their written response to the student's understanding or misconceptions, so the evidence of their responding skills for this student answer is lacking.

The results of the three parts of noticing were then compiled to create an overall picture of the noticing skills of the seven participants, as found in Table 9:

Table 9. Preservice Teachers' Skills in the Three Aspects of Noticing

Preservice Teacher	Attending	Interpreting	Responding
PA	Evidence	Robust evidence	Robust evidence
PB	Evidence	Limited evidence	Limited evidence
PC	Evidence	Robust evidence	Limited evidence
PD	Evidence	Robust evidence	Limited evidence
PE	Evidence	Limited evidence	Limited evidence
PF	Evidence	Limited evidence	Limited evidence
PG	Evidence	Limited evidence	Lack of evidence

Table 9 shows that one preservice teacher (PA) displayed sophisticated noticing skills in all three areas, and that one (PG) did not have satisfactory skills in any area beyond attending. Overall, this table shows that the skills of the preservice teachers for attending, interpreting, and responding could be generalized as "fair" or having evidence or limited evidence. However, the results varied according to participant, according to the aspect of noticing and according to the student answer to algebraic problem, as shown previously in tables 5 and 7. Although it is accurate to say that the participants struggled more with responding than with interpreting and with interpreting more than with attending, some participants struggled more than others to provide robust evidence for both interpreting and responding to student understanding and misconceptions. Also, as per the commentary on tables 5 and 7, no single preservice teacher was consistent in their responses for interpreting and responding across all nine stimuli; all had some level of variation, and two participants' responses varied wildly, making the categorization of their overall skill level difficult.

Discussion

All preservice teachers studied were able to describe mathematically important details in student answers, including the students' procedures in simplifying algebraic expressions. This is in keeping with Kılıç's (2019) findings that preservice teachers were able to display attending skills because they were able to state procedures and concepts used by students in general. Appova and Taylor (2019) also found that preservice teachers were able to identify the mathematical understandings grasped by students. However, some of the preservice teachers in this study struggled to provide in-depth explanations of the students' understanding, and most were too general in their plans to challenge the student's misconceptions and assist them to build correct understanding. Similarly, Zuya (2014) concluded that teachers' PCK was usually not sufficient for understanding and addressing students' misconceptions. As already noted in this paper, pedagogical content knowledge is necessary for teachers develop noticing skills, especially the second two facets of interpreting and responding. Merely attending to student understanding is not sufficient for teachers or prospective teachers because they should also be able to assess this understanding and act based on that assessment in order to express well-developed PCK. This is because a teacher with PCK grasps a fusion of both content and pedagogic knowledge and understands the teaching and learning process necessary for each aspect of the learning material (Setyaningrum et al., 2018). PCK therefore requires a deep understanding of the learning process and the individual student's learning needs. Preservice teacher's noticing skills should be developed beyond the level of attending, then, as a method of building their PCK and therefore preparing them for the classroom.

The seven preservice teachers' prior participation in the university's teaching practicum program seems to have been a helpful experience for them to draw on when providing comments on student strategies at the attending stage, and for some, at the interpreting stage as well. Due to their teaching experience, the participants were accustomed to being aware of the many facets of learning in the classroom, including those related to student thinking. These inferences are consistent with the results of the research by McDuffie et al. (2014), who found that the ability to understand students' ideas developed dynamically when preservice teachers underwent teaching experience. However, the participants studied did not have enough experience interpreting specific student understanding, which was obvious from the interviews and from the fact that the majority of participants experienced limitations in giving adequate interpretations. Participants admitted they had never conducted activities to analyze specific student understanding before. They had analyzed instructional videos according to general aspects of learning such as teacher-student interactions, but hadn't explicitly focused on student thinking processes. Hiebert et al. (2007) note that without structured training and experience in noticing, preservice teachers find it challenging to analyze specifics of student understanding. The findings regarding preservice teachers' interpreting skills in this study are consistent with previous ones which found that high sophistication of preservice teachers in attending to student ideas does not guarantee more sophisticated analyses or interpretations, and that novice teachers did not focus on interpreting student understanding (Jacobs et al., 2010). While the experience of the participants was helpful in building their noticing skills, it can be concluded that they did not have enough of the right experiences.

Regarding responding skills, the researchers found that the preservice teachers explained broad lesson plans or strategies to overcome student misconceptions without showing deep understanding through detailed explanations. Only one of seven was able to give concrete plans to assist in building correct understanding supported by specific examples, including testing of the wrong theory: a strategy to overcome misconceptions termed cognitive conflict. A misconception is a conception that differs from those of experts in that particular field and appears consistently in different contexts due to errors in the student's knowledge constructs. The main features, then, of the cognitive conflict strategy is asking students to observe a phenomenon that contradicts their misconceptions and guiding them to comprehend the correct concept (Parwati & Suharta, 2020). The results in Table 9 indicate that the vast majority of participants did not use the important details of student answers to interpret their understanding and plan future learning according to individual needs. This finding is similar to the research of Gotwals and Birmingham (2016), who determined that preservice teachers found it difficult to apply formative questioning techniques to uncover the nuances of individual student thought processes and respond in a constructive manner. Ding and Domínguez's (2016) research similarly revealed that the skills of preservice teachers in the three noticing aspects were inconsistent.

These results provide beneficial input for lecturers in teacher education programs in Aceh and wider Indonesia. From the holes in the participants' noticing skills, it is suggested that universities should pay attention to the building of preservice teacher noticing skills: specifically, preparing activities that help preservice teachers shift their attention to develop the skills to notice student thinking (Taylan, 2015), because this skill set can be learned (Jacobs et al., 2010; Stockero et al., 2020). University staff should include activities designed to build noticing skills in their curricula and assignments for each subject related to pedagogy. University teaching staff must also provide guidance and assistance so that preservice teachers understand more deeply the steps required and are able to put them into practice.

Conclusion

This study revealed that the seven participants' skills varied across the three aspects of noticing, with the overall picture being that most had only developed simple noticing skills. For the skill of paying attention, the preservice teachers demonstrated skills in understanding mathematical concepts, as well as good understanding and some analysis of strategies used in student answers to algebraic problems. This could be due to their experience in teaching practicums. However, the preservice teachers tended to have weaker interpreting skills, demonstrated by lack of detailed analysis of student answers. Many could only explain student answers in general terms, probably since they had not had specific or in depth training in interpreting student answers or student understanding. Finally, the participants as a whole displayed even lower skill levels in responding to student misconceptions, since most were only able to provide general plans for future learning. This is probably due to their lack of PCK, especially regarding how to respond to and reduce student misconceptions. These findings are in keeping with previous literature.

However, a unique finding of this study is the variation in evidence for preservice teacher skills, meaning that each participant had individual needs for improvement. One preservice teacher needed little or no further guidance in improving noticing skills, while another displayed a definite lack of skills indicating a need for more focused training on what noticing skills are and how to build them. This study has also been able to fill the gap in evidence somewhat regarding the level of preservice teacher noticing skills in Aceh and Indonesia. It also provides evidence as to the current level of effectiveness of the focus university's mathematics education program in building noticing skills and gives recommendations below for the improvement of this program's curriculum.

Recommendations

The researchers propose that noticing skills in preservice teachers should not just be improved by raising awareness in teacher education programs or by general guidance from university teaching staff, but more specifically, they should be built through continued, individualized training and mentoring to improve noticing skills, according to diagnostic assessment of preservice teachers' weaknesses and strengths in noticing. In the case of the specific university where the participants were studying, and possibly in other universities in Aceh and Indonesia, preservice teachers should be given diagnostic tests in noticing abilities, then given individualized tutoring or tutored in streamed groups with others of like skills. University staff who mentor individuals or lead groups should tailor the content to the university students, but the authors suggest that the learning activities could include: creating preservice teacher awareness of noticing skills; building PCK through appropriate learning activities to build pedagogical knowledge, pedagogical skills, mathematical content knowledge, knowledge of students as individuals, student thinking strategies and common student misconceptions; learning activities focused on analyzing student answers and misconceptions; role-play scenarios and simulations for preservice teachers to develop noticing skills through; video analysis; further diagnostic and formative assessment; and goal-making and evidence-collecting during practicum experience. Teaching staff must avoid simply giving information and instead provide feedback, guidance and assistance so that preservice teachers understand what noticing is in mathematics education and are able to practice it. It is recommended that this university's mathematics education program appoints a teacher educator to be responsible for the development of preservice teacher noticing skills.

It is recommended that future research on preservice teacher noticing skills in Indonesia encompasses a larger number of participants. Other suggested variables include a wider range of universities: for example, both state and private universities; universities in various provinces and islands; a comparison between the noticing skills of preservice and inservice teachers in typical Indonesian settings; and preservice teachers from various subject pedagogies. Observations of preservice teachers' actions with struggling students during practicums could also be analyzed, since hypothetical responses are to a degree abstracted, especially written ones. Finally, as the skills of the participating preservice teachers are studied, the curriculum of their teacher education program could also be analyzed and evaluated more details.

Limitations

The chief limitation of this study is the small number of research subjects. Also, they only come from one year level and one subject pedagogy major. However, the findings in this research are still useful and can be used as a basis for mathematics teacher educators to adjust curricula to improve preservice teachers' PCK and noticing skills.

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Author contribution statement

Johar: Conceptualization, writing (original draft), editing and visualization. Desy: Conceptualization, data collection, analysis, writing and editing. Ramli: Reviewing and editing. Sasalia: Analysis and editing. Walker: Editing and proofreading.

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