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The Influence of Mistake-Handling Activities on Mathematics Education: An Example of Definitions *

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Abstract: The study aims to find out the influence of Mistake-Handling Activities to determine mathematical definitions knowledge, which can be regarded as a component of mathematics content knowledge, of teachers on the development of teachers in providing mathematical definitions. Within this framework, Mistake-Handling Activities were carried out with five volunteer mathematics teachers. Written opinions and semi-structured face-to-face interviews were used as data collection tools. During the application, focus group interviews were carried out, and the application was enhanced with discussions. The data were analyzed using the document review method, and codes, categories, and themes were also determined. The results revealed that Mistake-Handling Activities yielded certain emotional advantages such as increasing teachers' interest and curiosity, critical thinking, self-confidence, awareness, and offering different viewpoints as well as yielding cognitive advantages such as recognizing their shortcomings, acknowledging the importance of knowing the definition of a concept, and using the definition.

Keywords: *Mistake-handling activities, definition, mathematics content knowledge, mathematics teacher.*

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Introduction

Negative knowledge is usually structured from mistakes and failures through trial-and-error and reflection, and it is mostly based on personal experiences (Akpinar & Akdogan, 2010). A positive knowledge-based teaching-learning environment, i.e. a teaching-learning environment based on the correct form of knowledge, may fall short; therefore, negative knowledge is regarded as a complement to positive knowledge (Heinze, 2005). Hence, negative knowledge-based Mistake-Handling Activities (MHA) are structuralism-based activities and argue that, for fully structured knowledge, mistakes and errors pertaining to a correct information should also be known as well as the correct information (Gedik & Konyalioglu, 2016).

Structuralism is an epistemological theory rather than a learning theory and aims creating true correct knowledge in an individual's own self and, here, mistakes are also instrumental to correct knowledge in building knowledge in its entirety (Gedik & Konyalioglu, 2016). Mistakes are ignored in behaviorist theories (Dalehefte, Seidel & Prenzel, 2012; Sandagata, 2005). In cognitive theories, emphasis is placed on misconceptions, whereas, in structuralism, mistakes are regarded as important opportunities for learning and as an unavoidable part of the learning environment (Dalehefte, Seidel & Prenzel, 2012; Santagata, 2005).

Errors, mistakes, and the perception of why they are wrong are an important part of the teaching-learning process (Konyalioglu, Aksu, Senel & Tortumlu, 2010). All of them will be able to provide the teachers to determine their students' thinking strategies and review their own knowledge and internalize the concepts in terms of both conceptually and procedurally. They also emphasize revealing the mistakes and the component of "the students' solution paths and understanding the student created logic" which makes the mistake-handling activities more important.

* This study is a part of the Ph.D. thesis of Solmaz Damla Gedik Altun.

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Literature Review

Some philosophers and scientists investigated the answer to "What is the role of mistakes in creating new knowledge? (Borasi, 1986; Harteis, Bauer & Gruber, 2008; Dalehefte, Seidel & Prenzel, 2012). The results led them to qualify mistakes as "learning-stimulating events" and as "critical knowledge".

To achieve correct perception in mathematics education, teaching methods focused on using mistakes as a bouncing board for mathematical inquiry (Borasi, 1986; 1989; 1994; 1996) through using conflictions and contradictions that suggest utilizing mistakes by "colliding contradictory concepts" (Movshovitz-Hadar & Hadass, 1990; Swan, 1983) were employed. Borasi (1986) carried out studies based on the belief that mathematical mistakes can be at least partially benefitted.

In their studies, Borasi (1989) reported that mistake analysis enabled children to develop better conceptual understanding through determining and clarifying conceptual mistakes, emphasizing new perspectives and considering unexpected elements. In addition, mistake analysis helped concretize the discussions of abstract subjects and allowed accessibility for students.

Studies on mistake and its use (Borasi, 1986; 1989; 1994; 1996; Heinze, 2005; Ginat, 2003), revealed that mistake learning can be accepted as a powerful tool to diagnose and remedy learning difficulties, provide motivation and a solid starting point for those who try to discover and explore their mistakes, induce interesting responses, and provide a new perspective for mathematical concepts and definitions. They also showed that investigating the role of mistakes in mathematical knowledge development can help elucidate mathematical knowledge production and provide a better approach to the limits, motivations, and features of mathematical methods. Furthermore, a more detailed investigation of mistakes can help acquire a more profound understanding of mathematics. In general, deliberation on certain mistakes was thought to help students realize the limits of mathematics and thereby call attention to the humanistic side of the discipline.

It is thought that in addition to the mathematics educators, the teacher candidates also will be able to more prepare for their lessons with the mistake-handling activities carried out in educational settings by taking into account the students' mistakes. Several researchers such as Radatz (1979) carried out different studies related to the analysis of student mistakes in mathematical settings. This study was carried out with the aim of diagnosing the causes of typical student mistakes. The mistake-handling activities led to several theories about the variety and the importance of the students' mistakes, the nature of the mathematical mistakes, the interpretation and ways of overcoming these errors. (Gagatsis & Kyriakides, 2000; Luo, 2004).

Research has revealed that teachers' thoughts on mathematics teaching and learning depend on their experience in learning mathematics (Fosnot, 1989; Skott, 2001). Thus, different behaviours of the teachers to the students' mistakes are caused from their preferences resisting to change. On the other hand, studies show that teachers' beliefs and ideas have a significant impact on their classroom applications. (Fang, 1996; Kagan, 1992). In this regard, Pajares (1992) points out that the teachers' beliefs and opinions affects their understanding, reasoning and their behaviours in classroom over time. Fang (1996) stated that understanding the teachers' idea and belief systems will contribute the quality of the education that they will effort on. In this context, revealing the ideas on students' mistakes will contribute on how they respond when they confronted with a student mistake. Also, this situation will provide sufficient information about the efficiencies of pedagogical education provided by the education faculties.

It was generally identified student mistakes and the teachers' approaches towards these mistakes in the literature. In some of the countries, mistake handling activities were carried out, and it was revealed that these activities had a positive impact on learning. However, it was identified that the teachers see the mistakes in a negative way. In this regard, there is a few researches on seeing the mistakes positively in the courses related to the pedagogical content knowledge and mathematical content knowledge in the education faculties. Because of this reason, it has come to mind such a study could be carried out.

Methodology

In the study, the case study approach was adopted as a qualitative research method. The case study method is preferred in human relations studies because of its realistic, applicable, and noteworthy properties (Brown, 2008), and they are based on revealing and thoroughly investigating individuals' way of thinking and comprehending different situations and phenomena.

The study was implemented in three phases. During the first phase, it was aimed that how the teacher candidates see themselves on making definitions. To this aim, a written opinion test related to making definitions was used as a preliminary diagnosis test, and the interviews were conducted to further elaborate. In the second phase, a specific day was determined for the participation to the study. Mistake-Handling Activities were shared with the teachers. Then, focus group discussions were conducted on the activities. At this stage, Mistake-Handling Activities were carried out every week for 3 hours and recorded on video. During the second phase, it was aimed to identify mistake handling activities caused which type of cognitive and affective differences on teacher candidates. During the third phase, it was aimed to identify that how teachers see themselves on making definitions. In this way, it was tried to determine how mistake handling activities change the secondary mathematics teachers' abilities of making definition.

Study Group and Data Collection

Homogenous sampling and convenience sampling methods were used as the sampling methods in the study. The aim in the homogenous sampling is to create a sub-group that the study could be carried out through creating a small homogenous sample (Yildirim & Simsek, 2011). Five of the individuals of sample were high school mathematics teachers. In terms of the convenience, these teachers were also selected as PhD students in terms of providing the study being quicker and practical. Thus, convenience sampling method was the secondary sampling method of the study.

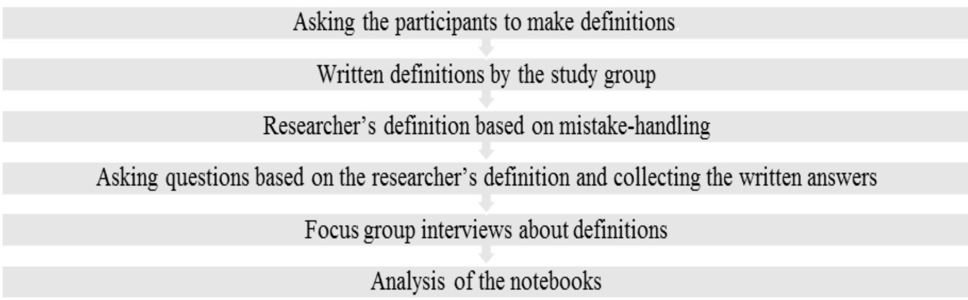
The study group comprises five high-school mathematics teachers working in five different cities. Data collection on definitions was carried out in certain periods spread over nine weeks and two of these weeks were reserved for the focus groups on definitions. The semi-structured interviews with the participants were carried out when an individual participant was available; focus group interviews were carried out when all the participants were available. The questions directed to the group during the focus group discussions are the following:

- Is this definition incorrect or correct?
- Is there a mistake in the statement?
- If there is a mistake, where did they make the mistake? How can it be fixed?
- What do you think people who made the definition(s) considered while making the definition(s)?
- Why did they make such a mistake?
- How would you fix it?
- What is/are the crucial point(s) and must-known concept(s) for the definition? Can you briefly explain these points and concepts? What should the student know and pay attention to?

The study was carried out in three stages. At the first stage, teachers' opinions on their ability to provide definitions were assessed using the case study method. The test to receive the written opinions of the teachers on their ability to provide definitions was applied as a pre-diagnosis test, and interviews were carried out.

At the second stage, a simple experimental study was conducted. During the experimental study, Mistake-Handling Activities were applied, and focus group interviews on the activities were carried out and recorded on video. Moreover, from the start of the activities, notebooks were given to the teachers in the study group, and they were asked to write their views on the MHA applications in the notebooks. The main purpose of these activities is, rather than immediately finding the correct answer, determining what kind of cognitive and emotional changes these activities cause.

The application cycle in the second stage is as follows:



At the third stage, the pre-diagnosis test to determine how teachers view their abilities to provide definitions was repeated as a final diagnosis test to determine the change mistake-handling activities cause in secondary education mathematics teachers' ability to provide definitions.

Analyzing the Data

The answers to the open-ended questions, which are included in the pre-test and post-test, were analyzed descriptively. In the descriptive analyses, the data obtained were summarized and interpreted according to the existing themes, categories, and codes. In this study, the analyses of the data collection tool was analyzed according to the pre-existing mathematics field information components in the literature. These components are stated as follows:

- To know the general situations of knowing and using the quantifiers, symbols, and conjunctions
- To know general definitions about knowing and using definitions
- General facts about knowing and using the theorem statements

- To know the proof methods to prove the general situation
- General situations related to solving questions
- General information about preparing questions
- General information about the to identify the mistake and to explain the cause of the mistake
- General information about using verbal and symbolic language
- General information about in-depth understanding of basic mathematics

Semi-structured interviews were conducted with the participants to further elaborate the questions asked in the final diagnostic test. The interviews were recorded on the voice recorder. The interviews were then written down and analyzed. The answers to the questions were subjected to descriptive analyses according to the predetermined field information components. In the process of analyzing the research data, the answers of the teachers to each question were arranged according to the aims of the research.

Focus group interviews were conducted on a voluntary basis with the participants to see the effect of Mistake-Handling Activities in the field of mathematics. The data obtained from these interviews were analyzed in detail for each participant according to the mathematical content knowledge components according to descriptive data analyses. Each word was carefully read and the explanations about the given operations wer carefully examined. The answers given to the questions have been interpreted, and the results were given in the findings according to the mathematical content knowledge.

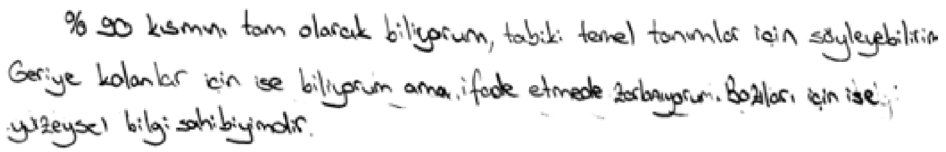
Findings / Results

Results on Pre-MHA Definitions

A pre-diagnosis test was applied to the teachers in order to identify how they see themselves on making definitions as a written opinion. Then, face-to-face interviews were carried out to make the process detailed. At this point, teachers' opinions related to making definitions before the MHA application were as follows:

Written opinions of the teachers on their ability to provide definitions were collected to reveal their views on definitions before the study. The views of the teachers on their ability to provide definitions prior to the MHS applications were given below.

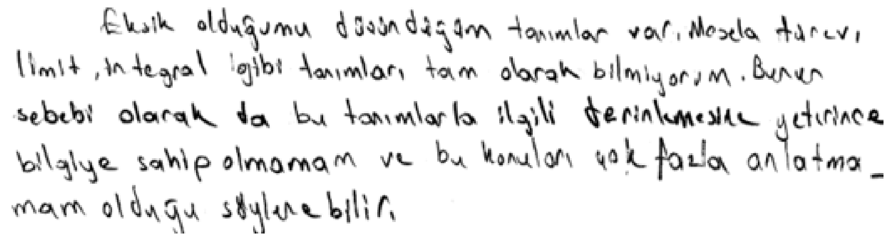
Of the teachers in the study group, two (A, N) claimed that they knew 80% of the basic mathematical definitions in secondary education curriculum, one teacher (Z) claimed that they roughly knew approximately 85% of the definitions, and one (S) claimed that they roughly knew approximately 60% of the definitions, while one teacher (M) claimed that they fully knew approximately 90% of the definitions and knew the rest on a superficial level.


%90 kısmını tam olarak biliyorum, tabiki temel tanımlar için söyleyebilirim. Geriye kalanlar için ise biliyorum ama ifade etmede zorlanıyorum. Bazıları için ise yüzeysel bilgi sahibiyimdir.

"I fully know 90% of the definitions, but this only applies to basic definitions. For the rest, I have knowledge, but I have hard time in explaining them. I suppose I only have superficial knowledge on others."

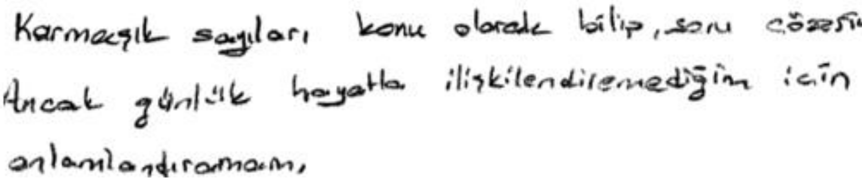
Figure 1. The written statement of Teacher Mahir on knowing the definitions

All the teachers in the study group stated that they made some incomplete definitions and attributed this to not explaining the definitions in their classrooms. One teacher (M) attributed incomplete definitions to not doing sufficient research. All the teachers in the study group stated that there were definitions they could not comprehend, such as, series, limits, and convergence.


Eksik olduğumu düşündüğüm tanımlar var. Mesela türev, limit, integral gibi tanımları tam olarak bilmiyorum. Bunun sebebi olarak da bu tanımlarla ilgili derinlemesine yeterince bilgiye sahip olmamam ve bu konuları çok fazla anlatmamam olduğu söylenebilir.

"There are definitions for which I think I am insufficient. For example, I do not exactly know the definitions of derivatives, limits, and integrals. The reason is maybe lacking in thorough and sufficient knowledge on these definitions and not teaching these subjects that much."

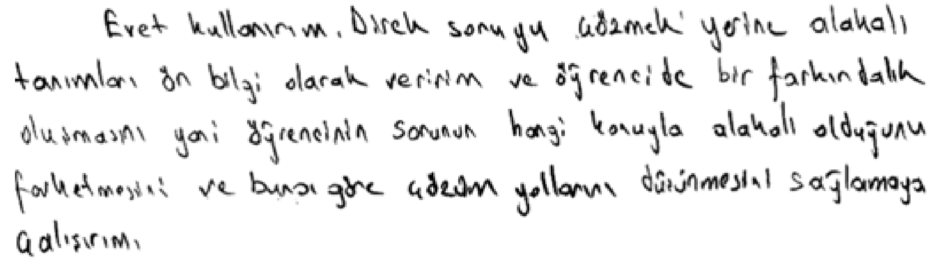
Figure 2. The written statement of Teacher Suat on fully knowing the definitions


Karmaşık sayıları konu olarak bilip, soru çözerim.
Ancak günlük hayatla ilişkilendiremediğim için
anlamlandıramam,

"I know complex numbers as a subject and can solve the questions. However, I cannot comprehend them because I cannot associate them with daily life."

Figure 3. The written statement of Teacher Zuhal on fully knowing the definitions

All the teachers stated that they used definitions in solutions, and three teachers (Z, A, N) said that they also used definitions in proofs. Furthermore, the teachers regarded definitions as the building blocks of mathematics and the foundation of mathematical knowledge.


Evet kullanırım. Direk soruyu çözmek yerine alakalı
tanımları ön bilgi olarak veririm ve öğrencide bir farkındalık
oluşmasını yani öğrencinin sorunun hangi konuyla alakalı olduğunu
farketmesini ve buna göre çözüm yollarını düşünmesini sağlamaya
çalışırım.

"Yes, I use them. Instead of directly solving a question, I first give the relevant definitions as a preliminary information and try to create awareness in the student, i.e. urge the student to realize to which subject the question is related and accordingly think of solutions."

Figure 4. The written statement of Teacher Suat on using definitions

With the questions directed at the teachers prior to the MHA, the written opinions of the teachers revealed that teachers were confident in providing and using definitions in general terms and view themselves competent in this respect.

Based on the written opinions and the face-to-face interviews before the MHA, it was found that teachers were self-confident on making definitions, and they see themselves sufficient in this context.

Results on the MHA Process

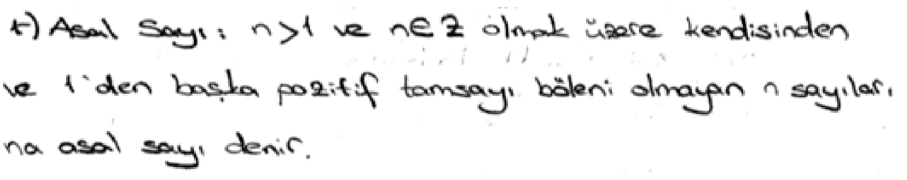
After receiving the views of teachers on definitions, we proceeded to the MHA applications. During this process, collecting written opinions on questions during a course, focus group interviews, and document analyses of the diaries after a week were performed. Teachers were first asked questions about the concept of definition, and their written opinions in this regard were collected. The answers of teachers during the application to some questions were given below.

The first definition the teachers were asked to provide was the definition of prime numbers. Only one teacher (M) gave the correct answer and four teachers failed to provide a full definition and left out certain points.


Asal Sayı= Kendisi ve 1 den başka böleni olmayan
sayı. 1 sayısı dışında

"Prime number= A number that has no divisors other than itself and 1. Except for 1."

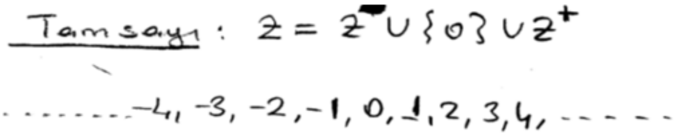
Figure 5. Teacher Asli's definition of prime numbers


f) Asal Sayı: n>1 ve n∈Z olmak üzere kendisinden
ve 1'den başka pozitif tamsayı böleni olmayan n sayılara
asal sayı denir.

"Prime number: with $n>1$ and $n\in Z$, n numbers that have no positive whole number divisors other than themselves and 1."

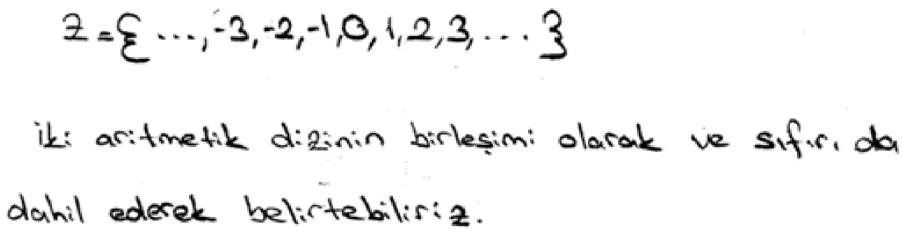
Figure 6. Teacher Mahir's definition of prime numbers

In the second question, teachers were asked to define whole numbers. The teachers failed to define whole numbers and all of the teachers tried to define whole numbers by using sets of numbers.



"Whole number: Z =Z …"

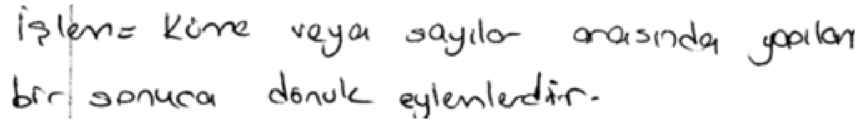
Figure 7. Teacher Naci's definition of whole numbers



"We can define it as the union of two arithmetic progressions with also including zero."

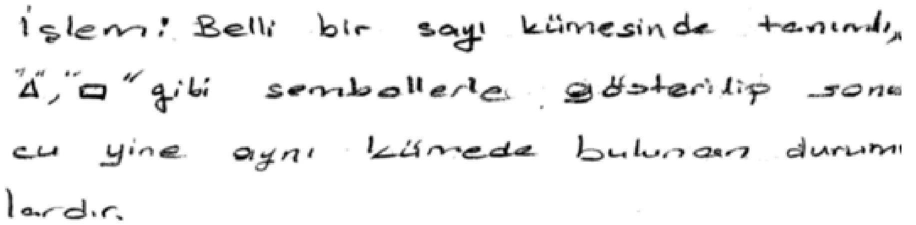
Figure 8. Teacher Mahir's definition of whole numbers

The third definition was the definition of operations. One teacher (N), albeit incomplete, tried to define operations with the concepts of relations and functions, and one teacher (Z) mentioned the closure property to define operations, while the rest of the group used examples to define operations.



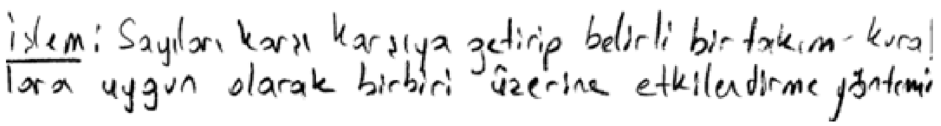
"Operation= Result-oriented actions made between sets or numbers."

Figure 9. Teacher Asli's definition of operations



"Operation: States defined in a certain set of numbers and represented with symbols such as, "Δ", and "□" whose results are in the same set."

Figure 10. Teacher Zuhal's definition of operations



"Operations: aligning numbers and then inducing their effects on each other by abiding by certain rules"

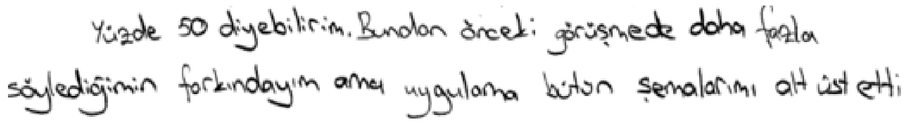
Figure 11. Teacher Suat's definition of operations

As the answers of teachers to the questions asked prior to MHA reveal, the teachers failed to fully provide the three definitions.

After receiving the written opinions at the beginning of MHA, we proceeded with the focus group interviews during the MHA process. The above questions asked at the beginning of MHA were opened up for group discussions.

Results After the MHA Applications

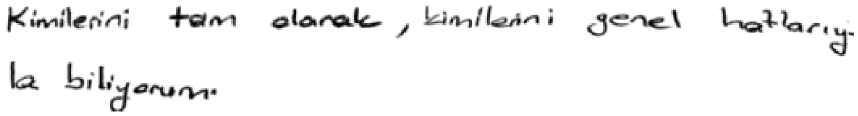
After the application, the teachers in the study group stated that they realized they previously overestimated the ratios, and they only knew, on average, approximately 50 to 60% of the basic mathematical definitions in secondary school curriculum in general terms, and they knew the rest only on a superficial level.



"I can say that I know 50% (of the definitions). I realize that I claimed more in the previous interview, but the application turned all my schemas upside down"

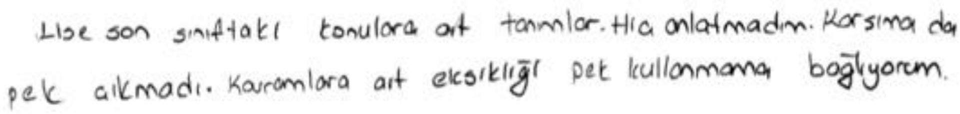
Figure 12. The written statement of Teacher Mahir on providing definitions

All the teachers in the study group stated that they still lacked in certain definitions, especially in those they do not use too often. In a similar manner, all of the teachers acknowledged that there still were definitions that they could not comprehend although they managed to comprehend some of the definitions during the application. All the teachers in the study group said that there were definitions that they cannot comprehend, such as, the definitions of series, limits, and convergence.



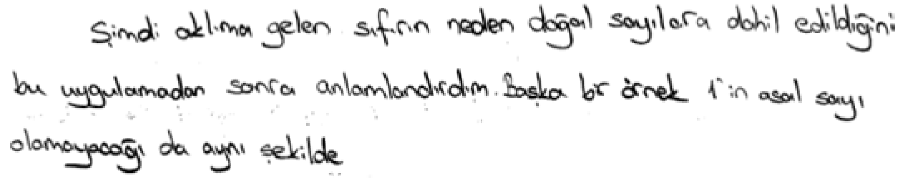
"I know some of them fully and some in general terms"

Figure 13. The written statement of Teacher Zuhal on providing definitions



"Definitions related to the course subjects in the senior year… I never taught them and never quite came across them. I attribute my lack in concepts to not quite using them"

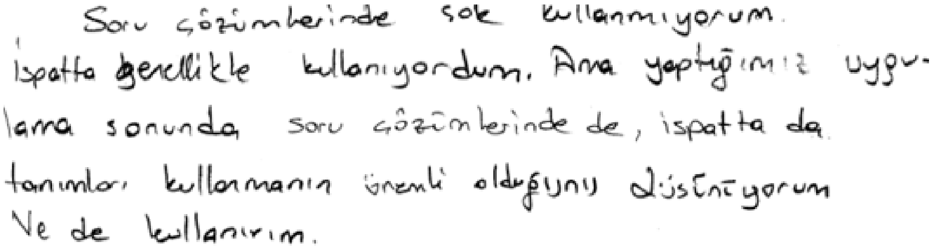
Figure 14. The written statement of Teacher Asli on providing definitions



"Now that I think of it, I understood why zero was included in natural numbers only after the application. Same goes for why '1' cannot be a prime number"

Figure 15. The written statement of Teacher Mahir on providing definitions

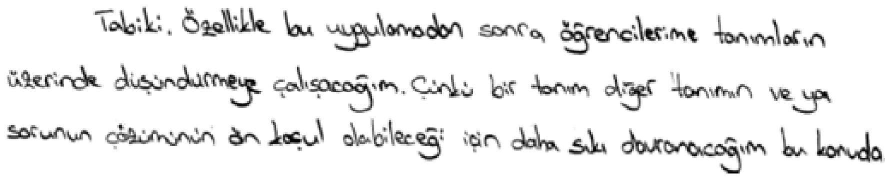
All of the teachers stated that they realized the importance of definitions in solutions and proofs and they will start to use definitions both in solutions and in proofs henceforward.



"I don't use them much in solutions. I usually use them in proofs. However, after the application we carried out, I think using definitions both in solutions and in proofs is important, and I will use them."

Figure 16. The written statement of Teacher Naci on providing definitions

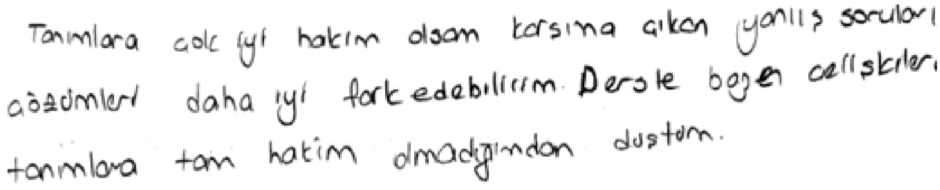
The teachers stated that they realized the importance of knowing the definitions especially after the application, and one teacher (M) said that they will also try to enable their students to comprehend the definitions. One teacher (A) made a remark that as definitions are better known, mistakes will be better noticed and solutions will be better understood. Two teachers (S, Z) regarded definitions as important parts of explaining the subjects and as building blocks of mathematics. Moreover, the teachers referred to definitions as the building blocks of mathematics and the foundation of mathematical knowledge.



Tabiki. Özellikle bu uygulamadan sonra öğrencilerime tanımların üzerinde düşündürmeye çalışacağım. Çünkü bir tanım diğer tanımın ve ya sorunun çözümünün ön koşul olabileceği için daha sıkı davranacağım bu konuda.

"Of course. Especially after this application, I will try to urge my students to think about definitions. I will lay emphasis on this issue more often because a definition may be a prerequisite of another definition or a solution."

Figure 17. The written statement of Teacher Suat on providing definitions



Tanımlara çok iyi hakim olsam karşıma çıkan yanlış soruları çözümleri daha iyi fark edebilirim. Derste bazen çelişkiler tanımlara tam hakim olmadığımdan düştüm.

"If I had a good understanding of definitions, I could better notice wrong questions and solutions. I sometimes felt conflicted in lectures because I didn't have a full understanding of definitions."

Figure 18. The written statement of Teacher Asli on providing definitions

Discussion and Conclusion

Definitions are the building blocks of mathematical thought and serve as a foundation for forming mathematical concepts, distinguishing different concepts, or expressing mathematical thoughts (Cakiroglu, 2013). Furinghetti and Paola (2002) describe the definitions as a gate leading to the development of mathematics. In other words, definitions have an important function in communicating the meanings of mathematical concepts and in providing unity on meaning. From this point of view, they form the basis of the mathematical language providing the written and verbal communication in the teaching and learning process (Shir & Zaslavsky, 2001). In mathematics, defining a concept is not as easy as it is expected. Any sentence describing what is a concept in mathematics cannot be a definition. We do not make a definition by explaining the properties of a concept, we just describe the concept by doing this (De Villiers, 1998). In order to create a mathematical definition, it is essential to select the necessary and sufficient properties, i.e. descriptive properties, through logical inference from the properties of the concept (Govender & De Villers, 2002; Fujita & Jones, 2007). In mathematics education, definitions of concepts are important issues to be addressed. There may be difficulties encountered by mathematics educators when making definitions.

In this study, mathematics content knowledge of candidate teachers was investigated with respect to their knowledge on mathematical definitions and ability to provide definitions. Prior to the applications, during collecting the written opinions of the teachers on their self-evaluation for pre-diagnosis, majority of the teachers claimed that they fully knew most of the basic mathematical definitions in secondary school curriculum and stated that they knew some of the definitions only on a superficial level and their insufficiency in these definitions was due to not teaching the concepts concerning these definitions. Again, prior to the application, all of the teachers stated that there were definitions that they could not comprehend. During the application, teachers described the concepts they were inquired about rather than providing their definitions and failed to explain the omitted points in their definitions. In their study on definitions to which they referred as a bouncing board, Borasi (1986;1989) reported that although the study group recognized that there were mistakes in the given definitions, they failed to explain the reasons of the mistakes, and they also made mistakes in defining the same concepts. During and after the application, the teachers realized what extent they lack in providing definitions, and there were too many definitions that they did not know. On the contrary to their views before the application, they stated that they can fully provide only half of the definitions in secondary school curriculum. All of the teachers said that there still were definitions that they could not comprehend even after the application. The teachers stated that, prior to the application, they did not put much emphasis on teaching definitions; however, after the application, they said that they fully understood the importance of definitions and will start to encourage their students to comprehend the definitions as well.

Moreover, teachers also stated that, by better learning the definitions, mistakes can be better diagnosed and solutions can be better understood.

Teachers stated that they used definitions in question solving and proofs, and they regarded definitions as the building blocks of mathematics and as the foundation of mathematical knowledge.

The study time could be increased for this type of studies. Also, different type of mistake handling activities could be used in order to get clearer results.

Additionally, the results of providing teachers to gain different aspects and thinking differently shows that Mistake-Handling Activities could make impacts on individuals thinking procedures.

According to the results of this study, the importance given to the definitions in mathematics teaching is not sufficient. Definitions should be taught more carefully in mathematics education in education faculties, and the place of definitions in mathematics education should be emphasized.

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