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The Pedagogical Manifestations: A Driver of Teachers' Practices in Teaching Algebraic Equations

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Abstract: Mathematics teachers' instructional strategies lack in-depth knowledge of algebraic systems and hold misconceptions about solving two algebraic equations simultaneously. This study aimed to gain an in-depth analysis of teachers' knowledge and perceptions about the promotion of conceptual learning and effective teaching of algebraic equations. The main question was, 'How do junior secondary school mathematics teachers manifest their pedagogical practices when teaching algebraic equations? This article reports on a qualitative, underpinned by the knowledge quartet model study, that sought to explore how junior secondary school teachers' pedagogical practices manifested in the teaching of algebraic equations. Data were collected from observations, semi-structured interviews, and document analysis of two mathematics teachers purposely selected from two schools. The collected data were analysed using a statistical analysis software called Atlas-ti. (Version 8) and triangulated through thematic analysis. The study revealed that teachers' choices of representations, examples, and tasks used did not expose learners to hands-on activities that promote understanding and making connections from the underlying algebraic equation concepts. The study proposed Penta-Knowledge Collaborative Planning and Reflective Teaching and Learning Models to enable teachers to collaborate with their peers from the planning stage to lesson delivery reflecting on good practices and strategies for teaching algebraic equations.

Keywords: *Classroom practices, pedagogical practices, penta-knowledge collaborative planning, teacher-centered methods.*

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Introduction

Globally, there are myths held by mathematics teachers regarding why learners must learn algebra. Demme (2018) and Zegarell (2017) suggest that learners should learn algebra as it (i) enhances their computational speed when solving problems, (ii) acts as a building block for learning advanced mathematics, and (iii) helps them to critically evaluate work done in other disciplines. Algebraic equations are used to solve real-life situations in various fields and beyond. For example, algebraic equations can be interpreted and used as a universal language for understanding the world around us in terms of making well-thought-out health, economic and social decisions. As a result, the inclusion of algebraic equations becomes an important component of the school curriculum.

The research (Ko & Karadag, 2013; Noris et al., 2022; Pappano, 2012; Shah, 2012) emphasize that algebra is a powerful tool that helps learners address real-life situations through problem-solving skills. In addition, those researchers emphasize that mathematics teachers encounter challenges usually manifested when their teaching strategies fail to help learners to transit from arithmetic to algebraic equations or think in concrete form, in aligning algebraic equations with real-life situations. As a result, teachers must have a solid understanding of both the content and the instructional strategies and materials to support this goal (Carpenter et al., 1999; Walters, 2014). On the contrary, teachers' content knowledge (CK) has been found to be primarily procedural with limited actual CK (Black, 2008). This limitation has been reflected in learners' understanding of algebra which was found to be procedural as opposed to conceptual (Ahmat et al., 2022; Asquith et al., 2007; Ko & Karadag, 2013). However, some studies, for example, Star et al. (2015), have contributed to the strategies and best practices in algebra teaching and learning focusing on how to address learners' misconceptions and errors. These studies did not probe the teachers' thinking to gain an in-depth analysis of teachers' ways of knowing how to effectively teach algebraic equations to promote conceptual learning in learners. It was therefore important to undertake an in-depth analysis of learners' algebraic manipulation and teachers' pedagogical practices.

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Several studies (Baumert et al., 2010; Black, 2008; Naseer, 2016) have indicated that teachers possess poor CK and pedagogical content knowledge (PCK) in algebraic equations, which negatively affects learners' understanding of algebra. Botswana is no exception, for example, Moalosi (2008) asserts that mathematics teachers lack in-depth knowledge of algebraic systems and hold misconceptions about solving two algebraic equations together. In support, the Trends in International Mathematics and Science Study (TIMSS) study of 2011 and 2015, revealed that Botswana performed lower than comparative countries like United States of America and Singapore, suggesting that opportunities to learn algebraic concepts were either not effective or not provided to standard. For these reasons, this study intended to establish teachers' pedagogical classroom practices in teaching algebraic equations, with the view of developing a professional development model that would be used to improve classroom delivery for increased learners' mathematics achievement. This was done by identifying junior secondary school teachers' pedagogical practices regarding how they approach the teaching of algebraic equations concepts and why they expect learners to solve algebraic equations in a particular way. This study aimed to investigate how junior secondary mathematics teachers' pedagogical manifestations drive their practices in teaching algebraic equations. By understanding teachers' pedagogical practices, a teaching and learning model was developed to guide the knowledge required to teach essential skills for the effective learning of algebraic equations.

Literature Review

The related literature on teachers' theoretical and pedagogical practices in the teaching of algebra and conceptions and strategies considered the historical development of algebra, the concept of a variable, and the use of symbols have been instrumental in shaping the present algebra, especially in schools. According to Ronan (2012), modern algebra concerns itself with a general algebraic structure instead of rules and procedures for manipulating their individual elements. Additionally, Ellerton et al. (2017) argue that algebra entails the study of variables to understand the world around us through modelling real problems. This transformation advocates for algebra that connects symbols to signs with emphasis on the use of algebraic language that leads to generalisation.

Teachers' mathematical CK and PCK have been found to be critical in the development of teaching mathematical concepts for learners' conceptual understanding of the subject matter (Naseer, 2016; Patton, 2002) and lesson plans have been proven to be crucial for determining teachers' PCK. For example, a study by Naseer (2016) on the algebraic CK and PCK of mathematics teachers established that the teachers did not create lesson plans and that one novice teacher had a scanty lesson done two years previously when he joined the profession. The lesson plan was characterised by teachers dominating the classroom instruction with learners passively receiving content. Furthermore, Naseer found that the teachers felt that they were experienced hence they used a textbook in place of a lesson plan. In the same way, lesson notes were copied directly from textbooks, suggesting over-dependence on the prescribed textbooks. This finding fits in with this research since the teachers observed were found to be teaching without lesson plans. The findings of this study established that the learners displayed many misconceptions and failed to apply what they were taught to new situations. This finding suggests that the use of a lesson plan is important as it can give an informed impression about whether teachers are deficient in algebraic content and lack the pedagogical knowledge for effective classroom instruction. Several studies (Koency & Swanson, 2000; Tajudin & Kadir, 2014) have also established that teachers' lack of PCK affects learners' conceptual understanding of algebraic equations as well as the development of misconceptions. In support, a study by Ghanaguru et al. (2013) that intended to link theory and practice among Malaysian teachers established that the planning and execution of lessons provided an assessment level of their pedagogical knowledge as the application of knowledge learnt from teacher training is revealed. It was further established that lesson planning and preparations are dependent on the experience of past successes and failures in terms of teaching styles and strategies. Ní Shúilleabháin (2015) corroborates as he established that teachers' PCK is enhanced through teachers' lesson planning and reflection on their teaching, which are features of their knowledge of content and students and knowledge of content and teaching. This result provided a lens of analysis for this study as failure to use lesson plans by mathematics teachers observed revealed that they were unable to reflect on learners' misconceptions and mistakes, suggesting their lack of both knowledge of content, and knowledge of learners and teaching (Ball et al., 2008). Further research (Boikhutso, 2010; Ghanaguru et al., 2013) in support emphasize the importance of lesson plans as a mechanism for checking teachers' effectiveness and competencies during lesson delivery.

It is worth noting that Botswana like other countries that value the role of algebra in society recognises the role played by algebraic equations in the school curriculum stressed through an aim requiring that learners should be able to handle and manipulate the symbolic mathematical language, algebra, and to use formulae and equations in problem-solving (Ministry of Education, 2010).

This as a result suggests the importance of algebra in the school curriculum as a driver to equip learners with skills to address real-life situations. For example, Hageraats (2016) underscored that the cost analysis of a business is done by setting margins and doing algebraic calculations, which lead to an analysis of the whole business structure. This analysis suggests that effective and efficient teaching of algebra empowers future leaders with the relevant skills to drive the economy.

Haggstrom (2008) and Kullberg et al. (2017) report that teachers engaged in different strategies for teaching and learning algebra, thus, providing contrasting views regarding teachers' classroom practices. For example, in China, some teachers exclusively presented equations with two unknowns, while others with three or more unknowns suggest that teachers from the same country presented algebraic content differently. In the Czech Republic, one teacher taught equations in relation to learners' everyday life situations, while the other enhanced learners' existing knowledge through linking equations to arithmetic processes (Novotna & Hospesova, 2008).

Research in Africa has also revealed how teachers teach algebra during mathematics instruction. For example, Osei's (2006) study found that South African teachers' conceptual algebraic knowledge and understanding were weak and fragile. In Botswana, Moalosi's (2008) study found mathematics teachers lacking in-depth knowledge of algebraic systems as they even failed to identify the topics that require the application of algebraic systems. It was further found that they held misconceptions about algebraic systems, especially when solving two equations together. These findings are critical to this study as Botswana teachers' pedagogical practices were explored and aligned with practices elsewhere.

The knowledge quartet (KQ) by Turner and Rowland (2008) as shown in Figure 1 was chosen as a conceptual framework as this study was intended to interrogate mathematics teachers' pedagogical practices in the teaching of algebraic equations, human behavior, which is difficult to predict. Turner and Rowland (2008) define the KQ as a comprehensive tool for thinking about how subject knowledge comes into play in the classroom. As a result, the observed teacher classroom practices provided an in-depth understanding of the causes of those pedagogical classroom practices as opposed to an interpretation of those practices.

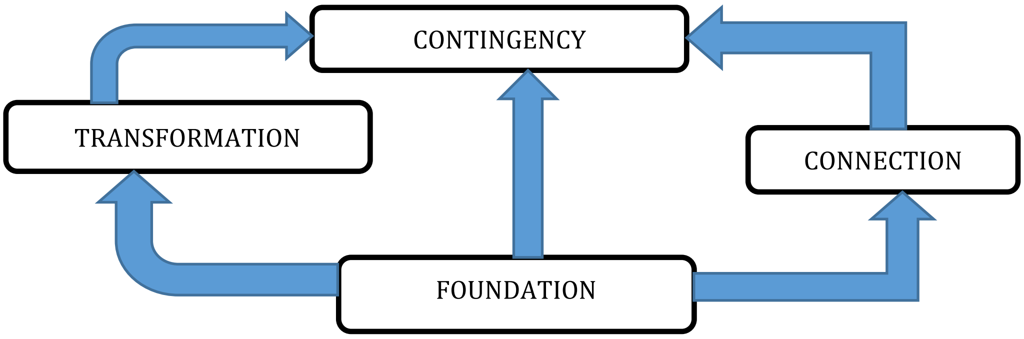

Figure 1. A Conceptual Framework for Mathematics Lesson Observation in View of Mathematics Teaching Development (Adapted from Turner & Rowland, 2008)

The KQ framework was designed for use in lesson observation and mathematics teaching development and to support focused reflection on the mathematical content of teaching to promote development in mathematical content and knowledge (Turner, 2012).

This study utilized the KQ to establish observed practices given how teachers draw from their knowledge of teaching mathematics and theoretical underpinnings during their lessons. In the same way, the KQ helped to identify the indicators of best or poor classroom practices as teachers were subjected to the framework in terms of their lesson planning and preparation, how they structured the lesson progression, how they reflected on learners' mistakes, and how they linked related concepts. According to Turner and Rowland (2008), the KQ is made up of four dimensions: (1) Foundation (teachers' theoretical background acquired during the teacher preparation stage); (2) Transformation (teachers' ability to transform the CK they possess into pedagogically powerful forms); (3) Connection (teacher's ability to put together certain choices and decisions about a mathematical concept or procedure); and (4) Contingency (concerned with the teachers' response to anticipated classroom events).

Methodology

Research Design

This article reports specifically on the data collected in a bigger study that aimed to explore junior secondary school teachers' pedagogical practices in teaching Form 2 algebraic equations in four schools in Botswana. The study was located within the interpretive paradigm which seeks to understand the situation from the perspective of the participants (Ary et al., 2010). It followed a qualitative research approach in a case study design that sought to explain how mathematics teachers' pedagogical manifestations drive their practices in teaching algebraic equations. The case study design was ideal for a very limited number of individuals as the subjects of the study for better data management. A qualitative approach was chosen since the study explored a social or human problem involving an interpretative, naturalistic approach to the world (Roller & Lavrakas, 2015).

Data Collection

Data were collected using semi-structured interviews, classroom observations, and document analysis (Babbie & Mouton, 2011; Creswell & Creswell, 2017). The study employed a non-participant observation technique, which provided an opportunity for researchers to gain an in-depth understanding through a thorough analysis and interpretation of the data. The lesson observation guide was designed to capture and give focus to the interaction between the teachers and learners and reflections on algebraic teaching instructional practices. An interview guide was developed to collect data regarding the mathematics teachers' expertise in algebraic content and the instructional techniques employed for the effective teaching and learning of algebraic equations. The semi-structured interviews allowed for an active discussion with the interviewee (Rabionet, 2011). The interview guide also enabled researchers to establish the activities designed by teachers to help learners to master the algebraic concepts. Additionally, the results of the in-depth semi-structured interviews and non-participant observation hinged immensely on the contexts from which the data were collected. The guide also provided an opportunity for the teachers to reflect and suggest other models of teaching and learning algebraic equations. Document analysis was intended to establish an interaction between the type of content suggested by the curriculum blueprint, content planning, and suggested teaching and learning strategies for effective learning of algebraic equations. The documents analysed included i) Scheme books and lesson plan booklets to establish whether the suggested teaching methods and strategies were utilized during the lesson planning; ii) Learners' exercise books to identify the mistakes and misconceptions against the teachers' strategies; and iii) Prescribed textbooks to identify the type of examples used against examples and illustration used during the teaching and learning of algebraic equations.

Sample

Two male teachers were purposely selected from two schools from a population of 21 government junior secondary schools in one region in Botswana. They had a degree qualification in Mathematics Education, with teaching experience of 8 and 12 years, and aged between 30 and 40 years respectively. Teachers did common scheming, administered the same tests, and moved at the same pace in terms of syllabus coverage since they belonged to the same region. Common scheming entailed planning and organising teaching activities that would be used by each teacher in a particular region at almost the same time as the other. This criterion was relevant to achieving the aim of this study since the teacher's pedagogical practices in terms of mathematics were comparable irrespective of the school at which they taught.

Data Analysis

In this study, thematic analysis (Bell, 2014) that entailed a systematic process of studying, sorting, and arranging the raw data obtained during the lesson observations, was utilized. The data collected from the classroom observations and document analysis were coded through Qualitative Data Analysis software called ATLAS.ti version 8.0 (ATLAS.ti Scientific Software Development GmbH, 2022), while the data from the semi-structured interviews were transcribed manually and coded through the same software.

Findings / Results

In this study, pseudonyms were used for the two mathematics teachers, *Koro* from School A and *Rothwe* from School B. The two participants had varied teaching experience and their different exposure to school cultures probably had a bearing on their pedagogical classroom practices. These variations in teaching experience also suggest that the participants had varied perceptions about instructional strategies, classroom management, and interactions when teaching algebraic equations. The analysis of data brought about a theme that related to the pedagogical manifestations discussed under two categories: (i) Teacher-directed instruction; and (ii) Teachers' conceptions about the teaching of algebraic equations.

Teacher-directed instruction

Findings in this study revealed that the teachers portrayed the role of an instructor possessing all the requisite knowledge on the topic of algebraic equations and delivering information that they felt was relevant to learners through the chalk and talk approach. By so doing the teachers employed teacher-directed instructional methods, where they stood in front of a classroom and presented that information deemed suitable for the learners as shown in Figure 2.

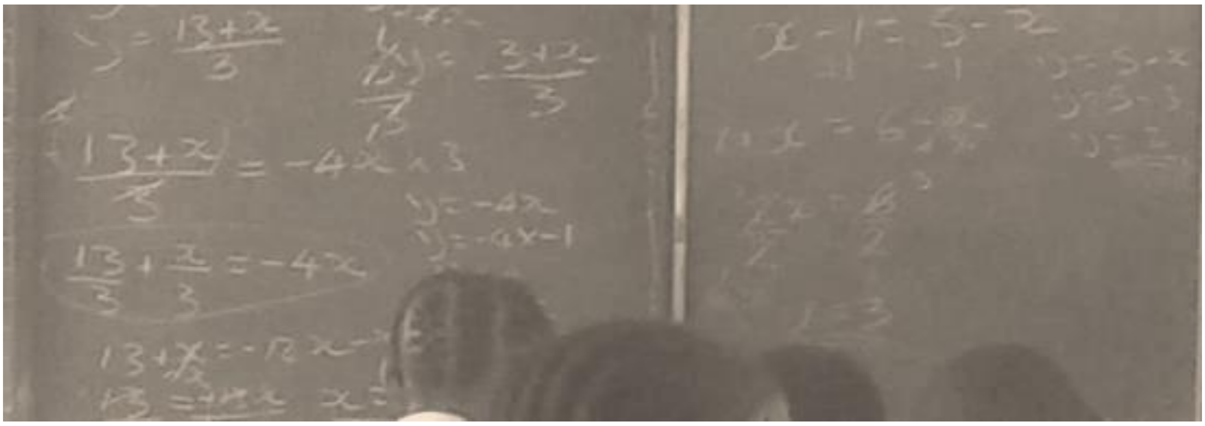

Figure 2. Koro Presenting Examples on the Chalkboard

It was also observed (see Figure 2) that *Koro* crafted these examples from his head and mostly presented them haphazardly not according to their level of difficulty intended to cater for learners of varied learning needs. This observation could suggest that he believed that picking unplanned questions from his head could command respect from his learners thus boosting his confidence in his role as the master teacher. However, unplanned questions can contribute to the omission of crucial concepts that could have been addressed when learners respond to well thought-crafted questions. Furthermore, the predominant teacher demonstration role ignored the fact that not all learners learn best when listening to the teacher and it is not always true that every concept is best taught through the chalk and talk method that dominated the instruction. *Koro*'s practice regarding his choice of examples is described by Ng and Dindyal (2016) as not merely choosing good examples but entails choosing examples leveraging on coherent example sets to build students' understanding to attain instructional goals. It could be noted that the teacher's ability to transform the CK he possessed was poor and not aligned to pedagogically powerful forms. The way mathematics teachers believed mathematics should be taught and assessed influences their pedagogical practices (Thompson, 1992), and additionally, teachers' prior experiences influence their classroom practice prescriptions (Ozgur & Aslan, 2016). As a result, mathematics teachers' prescribed way of teaching algebraic equations entailed their predominant use of plenty of examples to teach the algebraic equation concepts.

Teachers' Conceptions about the Teaching of Algebraic Equations

The lesson observation descriptions revealed that the two teachers' conceptions were inclined towards traditionalist than constructivist conceptions. The findings of the theme are presented in relation to two categories, (i) Step-by-step procedures; and (ii) Multiple strategies.

Step-by-Step Procedures

Observations prescribed that a learner must follow teachers' step-by-step procedures without digressing from the taught steps. For example, learners would be given an example for solving a linear equation and pick an appropriate mathematical skill that would manifest through a step-by-step procedure. In turn, a similar algebraic equation practice problem is given requiring similar algebraic manipulation, compelling learners to reproduce the availed step-by-step procedure. This is clear from the example that was presented by *Rothwe* when showing learners, a step-by-step procedure to solve the equation $\frac{2a}{7}+\frac{3a}{7}=1$ and started with an example of addition of a common fraction $\frac{2}{7}+\frac{3}{7}$ intended to illustrate the concept he wanted learners to master. He said:

Rothwe: *We are going to recap how to add these two fractions. How are you going to do it as your first step? Remember what you do here* $\frac{2}{7}+\frac{3}{7}$ *is the same as* $\frac{2a}{7}+\frac{3a}{7}$. *I want you to do this on your own now.*

After a few minutes of allowing learners to work on the fractional equation in their individual books, *Rothwe* continued to work out the problem on the chalkboard and explained the steps taken in the process. He wrote the equation, $\frac{2a}{7}+\frac{3a}{7}=1$ and then said:

Rothwe: *Step 1, we add the numerators and retain the common denominator. He continued to write* $\frac{(2a+3a)}{7}=1$. *What is supposed to be done after this?*

All learners in chorus: *Five aeeeee!*

Rothwe acknowledged the learners' response by writing the reduced equation as $\frac{5a}{7}=1$ and asked for the next step:

Rothwe: *What do we do next? Remember what we were doing under algebraic equations in variables appearing in one term only.*

Learner: *We multiply by 7 both sides.*

Rothwe: *Yeees! Very good and this becomes:* $\frac{5a}{7} \times 7 = 1 \times 7$ *which reduces to* $5a = 7$.

Throughout the process of solving the fractional equation, the demonstration structure entailed *Rothwe* writing on the chalkboard, then asking learners about the steps, followed by learners either in chorus or by a show of hands giving the correct step and this continued in a cyclic pattern. There was no instance when *Rothwe* asked a probing question to encourage deep thinking for learners' conceptual understanding. *Rothwe* continued asking:

Rothwe: *Then what is next, boys and girls?*

All learners in chorus: *Divide by five, siiiir!*

Rothwe: *Yes! Perfect! Divide by five and we get this:* $\frac{5a}{5} = \frac{7}{5}$ *reducing to* $a = \frac{7}{5}$ *and this is our solution.*

During interviews, we asked if he thought the emphasis on demonstrating step-by-step yields desired results and if other alternative methods of teaching could be used in teaching algebraic equations. His response was:

Rothwe: *To me like I said earlier demonstrating step-by-step method is the best as it saves time. The only problem is lack of practice, but I keep telling them that maths is naturally that it should be practiced daily. Hence, the method is not wrong in any way it's all about practice even if they know math, they should try one or two topics daily so that they don't forget.*

Probing further how *Rothwe* ensured that learners don't repeat mistakes made as they try to replicate the teacher's steps, his response was:

Rothwe: *It's all about emphasis, we state those possible mistakes, write them on the chalkboard and tell them to do that, do this, and with their experience over time, they will know how to avoid the mistakes.*

The constructivist conceptions were revealed through multiple strategies employed when solving algebraic equations. This observation was found to be a promising point of departure which when effectively utilized could help in the development of learners' conceptual knowledge of algebraic equations. The discussion of the conceptions follows.

Multiple Strategies

During the lesson observations, *Koro* provided learners with opportunities to explore the transpose and balance methods by showing them how the two methods work to give the same answer. *Koro* used the equation $\frac{x}{5} + 2 = 8$ to illustrate the two methods of solving linear algebraic equations. Learners were directed through the balance method following a demonstration in which he stressed that the balance method entailed using the reverse operations and asked:

Koro: *What signs are involved here, Thapelo?*

Thapelo: *Division and subtraction.*

Koro: *Karabo, now how do we use the division operation here? Remember we started off by saying there are two operations involved. We are done with the addition one, how do we deal with the division one?*

Karabo: *Multiply by five both sides.*

Koro: *Very good, inverse of division is multiply.*

Koro then multiplied by five on both sides and presented the equation as $5 \times \frac{x}{5} = 6 \times 5$ and continued to probe further on the steps to follow:

Koro: *Karabo*, what is the next step after multiplying by five both sides?

Karabo: *You cancel five into five and the answer will remain 11.*

Koro raised her voice showing disappointment with *Karabo*'s answer and remarked:

Koro: *Karabo is six times five equals 11 or 30?*

Karabo: *It's a mistake, Maa'aam, the answer, yes, is 30.*

Koro then continued to use another method which he termed 'crossing over the bridge' and this is what he said when introducing the method:

Koro: *Now, I am going to show you that you can kill a bird with one stone. I am going to show you another method which is a bit not mathematical, but it is used mostly in the teaching of algebraic equations. Let's use the same example and you will see that we get the same answer.*

He wrote the equation $\frac{x}{5}+2=8$ and likened the equal sign to the bridge by saying:

Koro: *We are going to take* $+2$ *to the other side of the bridge and the moment it crosses the bridge it becomes* -2*.*

Koro then wrote $\frac{x}{5}=8-2$ and said:

Koro: *Can you see that the presentation is different from the first method, here you don't subtract both sides but one side because the number* $+2$ *is 'crossing the bridge' and it will change its sign on the other side.*

During this time, when *Koro* was trying to explain the 'crossing the bridge' method to the learners, they looked at him with astonishment and none of them reacted to *Koro*'s comment on the differences between the two methods. He then proceeded, saying:

Koro: *This equation is then reduced to* $\frac{x}{5}=6$ *and here you have to be very careful as we are not multiplying both sides, but we are taking what we don't want from the left across the bridge. This means we take five, which we divided with across the bridge and once it crosses what happens? Yes, Gogo.*

Gogo: *We subtract 5, Ma'am*.

Koro: *How, Gogo? Can someone help Gogo.*

No one raised their hands and *Koro* then gave learners the answer:

Koro: *Remember, we divided by five on the left, once it crosses the bridge, we multiply the right side. Please remember what I said earlier on, you don't multiply both sides.*

Koro then presented the equation as $x=6\times 5$ and stated:

Koro: *This gives* $x=30$*, is it not the same as the first method?' it is, so now the choice is yours to choose the method you want. But like I said, this one is not mathematical, and I therefore instruct each one of you to use the first method always.*

It is evident from above that *Koro* introduced both methods not necessarily as a way of encouraging flexibility in the use of varied methods. He strictly adhered to the balance method and emphasized stressing the key aspects of the balance method in each step of solving linear algebraic equations. *Koro*'s notion of the balance method conforms to findings by Baratta (2011), who emphasizes the balance method as fundamental in developing learners' understanding of the concept of equality in algebra. For instance, when solving the equation $\frac{p}{4}-1=3$, *Koro* started by getting rid of the fraction by multiplying each term by four and this he clearly indicated in the step and emphasized verbally. *Koro* then employed the balanced method to get rid of -1 from the left-hand side, which was indicated by writing $+1$ on each side of the equal sign. When *Koro* was interviewed about why he discouraged learners from using the transpose method, he alluded to the idea that it is mathematically incorrect as the concept of balancing an equation is not reflected through the transpose method. In his own words, this is how *Koro* responded in the interview:

MR: *At some point, you kept on telling learners that the method of 'crossing the bridge' was not mathematical. Why was that so?*

Koro: *I am saying so because an equation has to be balanced from step 1 to the last stage of the answer. This method of 'crossing the bridge' is not acceptable as mathematically learners lose the concept of balancing through such methods.*

MR: *Don't you think there is a relationship between the method of taking a number to the other side and the balance method?*

Koro: *To be honest, the method of taking numbers to the other side of the equal sign is for people who are lazy and if you want to build a strong math foundation for your kids do the right thing. Like I said, the balance method relates well with see-saws that they have been playing with and this is the foundation of an equation, that, what you do on the left side you do it on the right side for it to be balanced and conforms to what an equation ought to be.*

Further probing resulted in *Koro* responding as follows:

MR: *How would you then justify solving linear algebraic equations using the two methods, which is not common among teachers? What were the reasons for that?*

Koro: *Of course, I know one may think it is time-wasting and I do agree with people who believe so because the syllabus is congested and there is barely any time to try many methods.*

MR: *Is it not more beneficial for learners to have a wide choice than worrying about different methods?*

Koro: *Different methods are okay for high achievers, but the low achievers will get more confused and what are the benefits of that. As teachers, we try to give methods that we know will make learners understand the procedures and steps needed in the examination.*

MR: *But this is what is prevailing, and the method seems to be giving results, why are you adamant that it is not mathematical?*

Koro: *To be honest this is a method that I used a lot in the past but since I went for further studies, when I came back, I realized that this method has the potential of causing misconceptions.*

MR: *How is this method a problem and causing misconceptions?*

Koro: *For example, this method of shortcut normally makes students change signs whenever they are rearranging terms on one side and one thing that is paining is that we have those teachers who use this method only and never give kids the right way of doing things. Teachers just encourage learners to use this shortcut method so that it will be easy for them to prepare for the examinations.*

Unfortunately, *Koro* had stereotyped conceptions regarding the use of the transpose method as he believed the method was likely to hamper learners' conceptual understanding of linear algebraic equations. There was no deliberate action taken by *Koro* to explore the challenges or effectiveness that are likely to be derived and benefitted from the method respectively. It is our profound belief that had *Koro* also emphasized the use of the transpose method as a way of providing learners with an opportunity to evaluate the pros and cons of using the two methods, he could have promoted logical reasoning among learners as it seems to be what characterises the transpose method. *Koro*'s conceptions about the use of multiple strategies in solving linear algebraic equations contradicts the views of Rittle-Johnson and Star (2007) who support the use of different solution methods as having the potential to act as a fundamental learning mechanism. In this way, learners can benefit from the pros and cons of the alternative methods of solving linear algebraic equations. In agreement, Silver et al. (2005) established that multiple solution methods are beneficial as learners can compare and reflect on interrelated key concepts.

Discussion

The results analysed revealed that *Koro* picked an appropriate mathematical skill that manifested through a step-by-step procedure demonstrated by solving a linear equation on the chalkboard. Such a practice was followed by giving learners a similar algebraic equation and the intended goal was to make learners proficient in answering examination questions. *Koro's* expectations were for learners to replicate the algebraic manipulation thus compelling learners to reproduce the step-by-step procedure. Further elaboration was revealed through *Koro's* emphasis on procedures of applying an algorithm, but the procedures seemed to be difficult for learners as they could not reproduce the procedures given to them. This result is compatible with the findings of Stipek et al. (2001) in which teachers gave learners work to practice newly introduced step-by-step procedures. It could be inferred that *Koro's* execution of step-by-step procedures followed an organised pattern, where one problem is shown to learners and a couple of similar problems are given as practice and this agrees with Rosenshine's (2012) description of drill and practice whereby new material is presented to learners in small steps with students given practice after every step. As a result, this organised practice by *Koro* allowed learners to improve their proficiency in the use of the procedure, critical for the final examination purposes as learners are expected to competently execute a variety of procedures across the topics. *Koro's* practice of presenting small amounts of algebraic problems and taking learners through practice support Groth (2017) in which learning procedures without understanding require extensive practice so as not to forget the steps. Consequently, this could be used to justify why *Koro* persisted with drill and practice regardless of learners showing a lack of understanding. The practice was persistently and consistently applied, with the belief that the reinforcement done throughout the years would give them practice and perfection to be proficient in the final examinations. This practice could further be likened to findings by Osei (2006) who found that student-teachers' conceptual knowledge and understanding of algebraic concepts were weak and fragile and contributed to learners' development of misconceptions (Tajudin & Kadir, 2014). This as a result could explain the generic trend among the two teachers who taught through rote learning.

The results analysis revealed that *Rothwe* derived a lot of joy every time the learners showed mastery of the steps that led to the correct answer. This was confirmed by the frequency of the learner reinforcement to correct answers, which coincided with the achievement of the different steps. However, we do not intend to imply that step-by-step procedures are ineffective, but how teachers execute the procedures is what matters. In support, Swanson (2017) has shown that for such an instructional approach to be effective, the teacher must tailor his instruction sequentially to guide learners through mastery of key concepts. The step-by-step approach was used during the introduction of every new concept of complex algebraic equations, thus implying that *Rothwe* ensured that learners master all the procedures required in tackling a variety of algebraic equations. The response was encouraging as learners were able to execute the procedures guided by the teacher who repeatedly asked what ought to be done at every step. In agreement, Star et al. (2015) underscored the importance of engaging learners in a discussion of each step of the incorrect answer to avoid future repetition of similar mistakes when they take their examinations.

An analysis of the interview with *Rothwe* indicates that his emphasis on step-by-step procedures was influenced by the national examination marking criteria that awarded marks for every correct step attempted by learners. This practice by *Rothwe* concurs with Mason (2003), who views step-by-step procedures as characterised by memorizing mathematical procedures, seen to be only good for examinations and useless otherwise as one memorizes rules and step-by-step procedures. This is a practice defined by Skemp (1964, 1976) as instrumental understanding and content focus by Kuhs and Ball (1986). Additionally, *Rothwe's* conceptualisation of teaching algebraic equations agrees with Saleh and Battisha (2020), who found that teachers employ step-by-step procedures because they want learners to proficiently complete mathematics tasks for examination purposes. Canobi (2009), Rittle-Johnson et al. (2001) and Rittle-Johnson et al. (2015) support this by likening *Rothwe's* conceptualisation of algebra teaching to learners' procedural knowledge of performing a series of steps to accomplish a specific goal. However, teaching learners through step-by-step procedures coupled with interrogation of each step by *Rothwe* empowered learners to recognise correct strategies for use in later problems independently and most importantly during their examinations.

The analysis revealed that *Rothwe* implicitly introduced the transpose method described by Hall (2002) as a change side-change sign technique. There was no mention of the term 'Transpose', but an analogy of the change sign technique was verbally stated without showing arrows of transposing to the other side of the equal sign. Avoiding the use of the term "Transpose" method by *Rothwe* could be motivated by the abstractness of algebra, hence relating it to "crossing the bridge" would be more exciting and ease the belief held about the difficulty of the subject. In this way, the intended benefits outweighed the mastering of the term "transpose" which learners effectively applied when solving an algebraic equation without having to remember the term "Transpose". Although the term itself was not necessarily a doorway to mastering solving algebraic equations, the key concepts of the implied transpose method were not explicitly stated as *Rothwe* emphasized in general terms that what happens to the term once it crosses the other side of the equal sign mattered most. In support, Hall (2002) notes that the transpose method tends to be the easy way out for learners as they do not have to worry about balancing the equation, but merely moving the existing quantities to the other side. Furthermore, *Rothwe* indirectly outlined the application of the concept of transposing in relation to the four basic operations and he stressed during the interview that 'transpose' to him did not add value to learners' understanding of solving algebraic equations. The finding implies that when moving quantities from one side to the other the four basic operations involved are reversed. *Rothwe's* explanation of the implied transpose method conforms to Buchbinder et al. (2015), who stress the movement of the inverse terms to the other side of the equal sign. Moreover, the analysis of the results showed that *Rothwe* preferred the transpose method over the balance method as he believed it was simple and a shortcut once learners have mastered it. Further analysis of *Rothwe* learners' class exercise books revealed variations in learners' presentations on the application of the implied transpose. There was a strong influence of *Rothwe's* presentation on both learners' work as one of the presentations was similar except for the omission of arrows. However, the arrows that were shown by one of the learners indicate a representation of *Rothwe's* verbal explanation of the implied transpose method in which he said, "when you have a term on the right side and take it to the left side, it must change its sign." It is worth noting that some learners deviated from the *Rothwe's* approach while others made sense of *Rothwe's* verbal explanation and introduced curved arrows to indicate the direction of movement.

The implied use of the transpose method is intended to reach the level of learners as the use of terminologies such as the inverse and transpose may not necessarily add value to the intended objective of changing the operation whenever a term is moved to the other side of the equal sign. The results analysis further revealed that *Rothwe* had no clue about the term transpose method but understood what it entails for one to use it in solving an algebraic equation. It was evident that his experiences with the use of the transpose method as a student during his secondary school days influenced his use of the method during the lesson observation. *Rothwe's* ignorance about the term "Transpose" could be justified by the fact that the mathematics examination does not emphasize definitions but calculations. What mattered most was the underlying properties necessary for the application of the method.

The Case of Koro

The results analysis showed that *Koro* used the balance and transpose method interchangeably and what mattered most was getting the correct answer. *Koro* had a strong influence on what the learners internalised as they believed and looked up to him for guidance as their role model. *Koro*'s mixed methods of solving algebraic equations do not conform to the description of solving equations using the transpose method as described by the 'Don't Memorise' blog. According to Don't Memorise, found in "*How do we Use the Transpose Method to Solve a Linear Equation?"* (Pai, 2014, 0:30-5:35), solving algebraic equations using the transpose method entails transferring a quantity to the other side by changing its sign and not performing an operation on both sides. Alternatively, in that blog, the balance method entails performing the same operation on both sides of the equal sign. The two methods lead to the same result but differ in terms of algebraic structure. For instance, the balance method employs the algebraic properties of equality (Cai et al., 2010) and it entails symbolic steps in a solution based on the algebraic properties of equality. The transpose method has no strict underlying algebraic manipulation and thus could be likened to a shortcut method for solving algebraic equations.

Koro did not display any indication of showing a distinction between the two methods in terms of what they entailed and how they are related. He applied the algebraic property of equality (subtraction) on both sides, a practice advocated by other researchers such as Kieran (1992) and Huntley and Terrell (2014) pointing out that learners are introduced to a variety of methods, including formal ones such as performing the same operation on both sides of the equation.

Furthermore, *Koro* unconsciously used a combination of the transpose and balance methods and loosely switched between the two, as his main concern was to employ simple strategies that would assist learners to understand solving algebraic equations. Furthermore, *Koro* believed the two methods were not different as they yielded the same results, and he was able to draw a thin line between the two in that the transpose method avoids the double writing of terms on both sides of the equation in every solving step, but keeps the equation balanced. It could be interpreted that *Koro* provided learners with an opportunity to explore the two methods at the same time allowing learners to switch and employ a convenient method in case the other method is not easily applicable. We believe that emphasizing the two methods in one problem could provide opportunities for learners to interrogate their relationships, thus paving the way to effective instructional practices and the meaningful learning of algebraic equations.

The analysis of the lesson observations, interviews and the learners' class exercise books showed that the observed teachers employed varied strategies which included balancing techniques and transposing. There were instances where a teacher would emphasize the balancing technique, while learners preferred the transposing method, suggesting that learners were able to analyse the two methods and decided on the one that they were comfortable with. Further analysis revealed extensive evidence of learners employing multiple strategies in solving algebraic equations, and this afforded them a choice to decide on the method they were comfortable with when solving algebraic equations. The use of multiple strategies such as a constructivist perspective entails providing learners with opportunities to solve mathematics problems in more than one way. This result agrees with Haggstrom (2008) and Kullberg et al. (2017) who reported that teachers engaged in different strategies for learners to meaningfully engage in the learning process, resulting in enhanced understanding of concepts.

The choice of representation of multiple strategies by teachers allowed learners to solve linear equations using two different methods suggesting an important aspect of teaching, i.e., that planning is critical in that it allows the teacher to reflect on different learning styles and take into cognisance the misconceptions that learners bring into the classroom. In this way, the teacher can plan how to guard against suggested misconceptions and as a result suggest suitable strategies. Reactions and response to learners' mistakes by the teachers was an indication that they employed reflective teaching in their practices. However, they were limited to effectively addressing learner misconceptions as the reflections were sporadic and not prior planned for. This finding corroborates Ball et al. (2008) who established that teachers were unable to reflect on learners' misconceptions and mistakes due to their failure to develop lesson plans in which they could strategise on how the planned instruction would be executed. Similarly, Boikhutso (2010), Ghanaguru et al. (2013) and Ní Shúilleabháin (2015) support the importance of lesson plans as a mechanism for checking teachers' effectiveness and competencies during lesson delivery.

It was, however, observed that the observed lessons provided some insights given improvements in support of the knowledge quartet. For example, the lesson plans allowed for the inclusion of the rationale of the lesson, teacher activity and learner activity components compelling teachers to consider the learners' immediate environment as they plan with the view of using materials known to learners for meaningful learning. As indicated earlier, teachers picked linear algebraic problems from their heads, suggesting that lesson planning was not thorough and as such a mismatch between lesson plans and lesson delivery was noted.

It is based on these observed classroom practices that the researchers proposed inclusion of collaborative planning and reflective teaching in view of the following aspects intended to compliment the four dimensions of the knowledge quartet: (a) Justification for teaching a concept; (b) Reflection of previous lesson or past experiences; (c) Possible learner misconceptions;(d) Possible strategies to address possible misconceptions; (e) Improvisation of teaching and learning materials; (f) Assessment of learning; (g) Linking content to learners' environment; and (h) Collaborative teaching.

The proposed inclusion of collaborative and reflective planning and teaching is hoped to enhance mathematics teachers' continual professional development. This implies that the focus will not only be anchored on the assessment of teaching but will also have aspects of assessing learning as learners' possible misconceptions and mistakes made during lessons will be used to plan for subsequent lessons. Furthermore, this fifth dimension is hoped to provide a supportive mechanism for producing a well-designed lesson plan that accommodates the KQ dimensions and ensures that teaching and learning are assessed with the view of addressing learners' misconceptions and promoting conceptual understanding of mathematics concepts. Figure 3 presents a modified Knowledge Quartet Model:

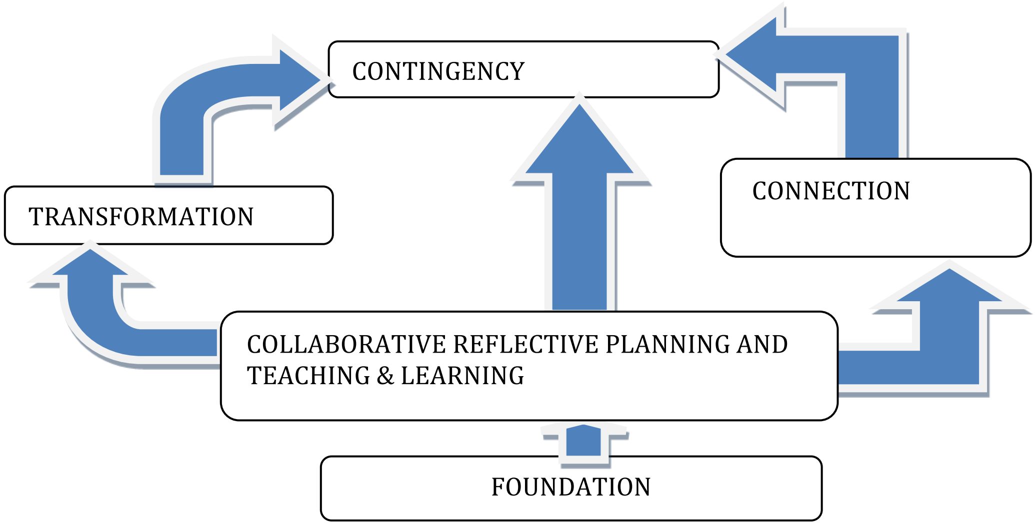

Figure 3. The Penta-Knowledge Collaborative Reflective Planning and Teaching & Learning Model (PKCPTL Model).

Conclusion

The purpose of this study was to explore junior secondary school mathematics teachers' pedagogical practices in the teaching of algebraic equations. The study established that both teachers predominantly used teacher-centred methods (drill and practice) and did not use lesson plans for any of the observed lessons. The study, in addition, established that the teachers avoided engaging in activity-based approaches as they feared that it was likely to affect syllabus coverage and not needed when solving examination questions.

Some of the observed teaching methods could be viewed as practices that could encourage active classroom discussions, but the observed practices were contradictory as teachers in most cases delivered all information that they felt was relevant to learners through chalk and talk approaches. Notably, mathematics teaching is a highly complex activity that must be acknowledged when teaching is analysed and discussed. The KQ, in its essence, assisted in the management of such complexities observed in the teaching and learning of algebraic equations. The KQ framework helped in understanding the contribution of teacher knowledge to the teaching of algebraic equations through the events enacted and observed in actual classrooms. This led to new knowledge where the *Penta-Knowledge Collaborative Reflective Planning and Teaching & Learning Model* (*PKCPTL*) was proposed for teacher professional development to address challenges encountered by teachers during classroom instructional practices. The *PKCPTL* Model is to enable teachers to collaborate with their peers within and neighboring schools from the planning stage to lesson delivery and to reflect on good practices and strategies for teaching algebraic equations.

Recommendations

Gaps were established in the form of a lack of lesson planning and teacher dominance that characterised classroom instructional practices in the region where the study was conducted. Based on the findings, the study recommends collaborative planning and teaching among mathematics teachers. This is based on the fact that mathematics teachers who were observed have been found to have weak teaching skills which adversely affected the teaching of algebraic equations. Furthermore, mathematics teachers should observe one another in order to improve and reflect on their classroom practices, consequently building a community of practice with neighboring schools. Through such communities of practice, mathematics teachers would develop activities intended to turn classroom teaching into activity-based learning. These are opportunities for capacity building of novice teachers to help them utilize their theoretical underpinnings to improve their PCK for effective lesson planning and delivery. Further research is also recommended to explore the effectiveness of the proposed PKCPTL model in mathematics classroom practices. This would provide useful information to enhance teacher professional development to influence a paradigm shift from teacher-centred approaches to activity-based learning.

Limitations

The scope of this study was limited to a few schools in one region. The research sample of two teachers from two schools was too small to generalize the findings to the rest of the schools in the region taking into consideration that there are some schools that are over 100km from the sampled schools. The fieldwork was limited to two interviews, two lesson observations for one teacher, and one lesson observation for the other as they taught through traditional methods covering a lot of materials in a short period. Secondly, all the lessons observed had no lesson plans and only adhered to the schedule for submitting lesson plan booklets to senior teachers which influenced teachers to retrospectively complete lesson plans for topics already taught. There was a recurring pattern in lessons observed across the teachers whereby teachers would demonstrate an example on the chalkboard, give learners work to replicate the procedures, and every time learners encountered challenges the teachers took over. There was no attempt to actively engage learners, thus making the structure of the presentation repetitive. It was, however, pointless to have more observations as there was nothing different likely to emerge, thus data saturation was reached after the second observation.

Authorship Contribution Statement

Salani: Conceptualization, design, analysis, writing, drafting the manuscript. Jojo: Editing/reviewing, supervision. mentorship, and critical revision of the manuscript.

References

Ahmat, N., Azmee, N. A., Mohamed, N. H., Zamzamir, Z., Zahari, N. S., Shafie, S., Mohamed, N. A., & Raja Ma'amor Shah, R. N. F. A. (2022). Knowledge, skills and attitude of pre-service mathematics teachers towards higher-order thinking skills. *International Journal of Educational Methodology, 8*(4), 795-804. https://doi.org/10.12973/ijem.8.4.795

Ary, D., Jacobs, L., Razavieh, A., & Sorensen, C. (2010). *Introduction for research in education* (8th ed.). Wadsworth Cengage Learning. https://cutt.ly/sNJFIry

Asquith, P., Stephens, A. C., Knuth, E. J., & Alibali, M. W. (2007). Middle school mathematics teachers' knowledge of learners' understanding of core algebraic concepts: Equal sign and variable. *Mathematical Thinking and Learning, 9*(3), 249-272. https://doi.org/10.1080/10986060701360910

ATLAS.ti Scientific Software Development GmbH. (2022). *Atlas.ti* [Computer software]. https://atlasti.com

Babbie, E., & Mouton, J. (2011). *The practice of social research*. University Press.

Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Baratta, C. (Ed.). (2011). *Environmentalism in the realm of science fiction and fantasy literature*. Cambridge Scholars Publishing.

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., & Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, *47*(1), 133-180. https://doi.org/10.3102/0002831209345157

Bell, E. (2014). Rethinking quality in qualitative research. *Australian Journal of Rural Health*, *22*(*3*), 90-91. https://doi.org/10.1111/ajr.12119

Black, C. M. (2008). Working for a healthier tomorrow*: Dame Carol Black's review of the health of Britain's working age population.* The Stationery Office.

Boikhutso, K. (2010). The theory into practice dilemma: Lesson planning challenges facing Botswana student-teachers. *Improving Schools*, *13*(3), 205-220. https://doi.org/10.1177/1365480206061994

Buchbinder, O., Chazan, D., & Fleming, E. (2015). Insights into the school mathematics tradition from solving linear equations. *For the Learning of Mathematics*, *35*(2), 2-8.

Cai, J., Nie, B., & Moyer, J. C. (2010). The teaching of equation solving: Approaches in standards-based and traditional curricula in the United States. *Pedagogies: An International Journal*, *5*(3), 170-186. https://doi.org/ghvb5j

Canobi, K. H. (2009). Concept–procedure interactions in children's addition and subtraction. *Journal of Experimental Child Psychology, 102*(2), 131-149. https://doi.org/10.1016/j.jecp.2008.07.008

Carpenter, T. P., Fennema, E., Franke, M. L. L., & Levi, L. (1999). *Children's mathematics: cognitively guided Instruction* (1st ed.). Heinemann.

Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches.* Sage Publications.

Demme, I. (2018, July 17). Six Reasons why we learn algebra. Demme Learning Blog. https://cutt.ly/JNJFX6R

Ellerton, N. F., Kanbir, S., & Clements, M. A. (2017). Historical perspectives on the purposes of school algebra, 40 years on: We are still learning! In A. Downton, S. Livy, & J. Hall (Eds.), *Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp.221–228). MERGA.

Ghanaguru, S., Nair, P., & Yong, C. (2013). Teacher trainers' beliefs in microteaching and lesson planning in a teacher training institution. *The English Teacher*, *42*(2), 104-116. https://cutt.ly/qNJFNNz

Groth, R. E. (2017). Classroom data analysis with the five strands of mathematica proficiency. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, *90*(3), 103-109. https://doi.org/jj7r

Hageraats, E. (2016). *Using relation algebra to generate feedback to learners in an intelligent tutoring system for business rules* [Unpublished Master's thesis]. Open Universiteit Nederland.

Haggstrom, J. (2008). *Teaching systems of algebraic equations in Sweden and China: What is made possible to learn?* Acta Universitatis Gothoburgensis.

Hall, R. D. (2002). An analysis of errors made in the solution of simple linear equations. *Philosophy of Mathematics Education Journal*, *15*(1), 1-67.

Huntley, M. A., & Terrell, M. S. (2014). One-step and multi-step linear equations: A content analysis of five textbook series. *ZDM, 46*, 751–766. https://doi.org/10.1007/s11858-014-0627-6

Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the national council of teachers of mathematics* (pp. 390-419). Macmillan Publishing Company.

Ko, Y.-Y., & Karadag, Z. (2013). Fostering middle school learners' relational thinking of the equal sign using GeoGebra. *Mevlana International Journal of Education, 3*(3), 45-49.

Koency, G., & Swanson, H. L. (2000, April 24-28). The special case of mathematics: Insufficient content knowledge a major obstacle to reform. [Paper presentation]. *American Educational Research Association Conference* 2000, New Orleans, LA.

Kuhs, T., & Ball, D. L. (1986). Approaches to teaching mathematics: Mapping the domains of knowledge, skills, and dispositions. *National Center for Research on Teacher Education*, Michigan State University.

Kullberg, A., Kempe, U. R., & Marton, F. (2017). What is made possible to learn when using the variation theory of learning in teaching mathematics? *ZDM Mathematics Education*, *49*(4), 559–569. https://doi.org/f94dsc

Mason, L. (2003). High school students' beliefs about maths, mathematical problem solving, and their achievement in maths: A cross-sectional study. *Educational Psychology, 23*(1), 73-85. https://doi.org/10.1080/01443410303216

Ministry of Education. (2010). Three-year junior certificate syllabus*.* Curriculum Division.

Moalosi, S. S. (2008). An investigation of teachers' subject matter knowledge and pedagogical content knowledge in Algebra: Study of Botswana junior secondary schools. [*Unpublished Master's Thesis*]. University of Botswana.

Naseer, M. S. (2016). Algebraic content and pedagogical knowledge of sixth grade mathematics teachers [*Doctoral dissertation]*. Walden University. https://cutt.ly/aNJGrOY

Ng, L. K., & Dindyal, J. (2016). Examples in the teaching of mathematics: Teachers' perceptions. In P. C. Toh., & B. Kaur (Eds.), D*eveloping 21st century competencies in the mathematics classroom: Yearbook 2016, Association of Mathematics Educators* (pp. 189-211). World Scientific. https://doi.org/10.1142/9789813143623_0011

Ní Shúilleabháin, A. (2015). Developing mathematics teachers' pedagogical content knowledge through iterative cycles of lesson study*.* In K. Krainer & N. Vondrová (Eds.), *Proceedings of the Ninth Conference of the European Society for Research in Mathematics Education (CERME9, 4-8 February 2015)* (pp. 2734-2740). Charles University in Prague, Faculty of Education and ERME.

Novotna, J., & Hospesova, A. (2008). Types of linking in teaching algebraic equations. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education and PME NA XXX* (Vol. 4, pp. 49-56). https://cutt.ly/oNJGuGL

Noris, M., Saputro, S., & Muzazzinah. (2022). The virtual laboratory based on problem based learning to improve students' critical thinking skills. *European Journal of Mathematics and Science Education, 3*(1), 35-47. https://doi.org/10.12973/ejmse.3.1.35

Osei, C. M. (2006). Student teachers' knowledge and understanding of algebraic concepts: The case of colleges of Education in the Eastern Cape and Southern KwaZulu Natal, South Africa [*Unpublished Doctoral thesis].* University of the Witwatersrand.

Ozgur, I. O., & Aslan, M. (2016). Prescriptions guiding prospective teachers in teaching mathematics educational sciences: *Theory & Practice, 16*(3), 735-769. https://doi.org/10.12738/estp.2016.3.0371

Pai, G. (2014, December 20). How do we use the transpose method to solve a linear equation? [Video file]. YouTube. https://cutt.ly/3NJGadc

Pappano, L. (2012). The Algebra Problem. How to elicit algebraic thinking in learners before eighth grade. *Harvard Education Letter*, *28*(3)*,* 1-2. https://cutt.ly/rNJGgXW

Patton, M. Q. (2002). Two decades of developments in qualitative inquiry: A personal, experiential perspective. *Qualitative Social Work, 1*(3), 261-283.

Rabionet, S. E. (2011). How I learned to design and conduct semi-structured interviews: An ongoing and continuous journey. *Qualitative Report, 16*(2), 63-566. https://doi.org/10.46743/2160-3715/2011.1070

Rittle-Johnson, B., Schneider, M., & Star, J. R. (2015). Not a one-way street: Bidirectional relations between procedural and conceptual knowledge of mathematics. *Educational Psychology Review*, *27*(4), 587-597. https://doi.org/10.1007/s10648-015-9302-x

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, *93*(2), 346-362. https://doi.org/10.1037/0022-0663.93.2.346

Rittle-Johnson, B., & Star, J. R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge? An experimental study on learning to solve equations. *Journal of Educational Psychology*, *99*(3), 561. https://doi.org/10.1037/0022-0663.99.3.561

Roller, M. R., & Lavrakas, P. J. (2015). *Applied qualitative research design: A total quality framework approach*. Guilford Publications.

Ronan, M. A. (2012). *Modern algebra. Encyclopedia Britannica.* https://www.britannica.com/science/modern algebra

Rosenshine, B. (2012). Principles of instruction: Research-based strategies that all teachers should know. *American Educator*, *36*(1), 12-39.

Saleh, M., & Battisha, M. (2020). A Proposed paradigm for the requirements for designing and using digital games-based learning by educable intellectual disabled children. *Technium Social Sciences Journal, 2(*1), 37–66. https://doi.org/10.47577/tssj.v2i1.5

Shah, S. (2012). Transition of students from arithmetic to algebra [*Unpublished master's dissertation*]. Aga Khan University.

Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C., & Strawhun, B. T. F. (2005). Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. *The Journal of Mathematical Behavior*, *24*(3-4), 287-301. https://doi.org/10.1016/j.jmathb.2005.09.009

Skemp, R. R. (1964). *Understanding mathematics*. University of London Press.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77(1), 20-26.

Star, J. R., Caronongan, P., Foegen, A., Furgeson, J., Keating, B., Larson, M. R., Lyskawa, J., McCallum, W. G., Porath, J., & Zbiek, R. M. (2015). *Teaching strategies for improving algebra knowledge in middle and high school learners* (NCEE 2014-4333). National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, U.S. Department of Education.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction*. Teaching and Teacher Education, 17*(2),213-226. https://doi.org/10.1016/S0742-051X(00)00052-4

Swanson, H. L. (2017). Searching for the Best Model for Instructing Students with Learning Disabilities. *Focus on Exceptional Children, 34*(2), 1-16. https://doi.org/10.17161/foec.v34i2.6785

Tajudin, N. A. M., & Kadir, N. Z. A. (2014). Technological pedagogical content knowledge and teaching practice of mathematics trainee teachers. In *AIP Conference Proceedings* (Vol. 1605, No. 1, pp. 734-739). American Institute of Physics. https://doi.org/10.1063/1.4887681

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 127–146). Macmillan Publishing Co, Inc.

Turner, F. (2012). Using the Knowledge Quartet to develop mathematics content knowledge: The role of reflection on professional development. *Research in Mathematics Education*, *14*(3), 253-271. https://doi.org/jj7s

Turner, F., & Rowland, T. (2008, March 18). The knowledge quartet: A means of developing and deepening mathematical knowledge in teaching*? Seminar presented at the Nuffield Seminar Series on Mathematical Knowledge in Teaching University of Cambridge*, UK. https://bit.ly/NuffieldSeminarSeries

Walters, K. (2014). *Professional development strategies to support student success in Algebra I*. American Institutes for Research.

Zegarell, M. (2017). *Basic math & pre-algebra workbook for dummies* (3rd ed.). John Wiley & Sons.