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The Effectiveness of Teaching Derivatives in Vietnamese High Schools Using APOS Theory and ACE Learning Cycle

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Abstract: The actions, processes, objects, and schemas (APOS) theory is a constructivist learning theory created by Dubinsky based on Piaget's epistemology and used to teach math worldwide. Especially the application of APOS theory to the curriculum of a mathematics class helps students better understand the concepts being taught, which in turn contributes to the formation and development of mathematical competencies. With the aid of the APOS theory and the activity, classroom discussion, and exercise (ACE) learning cycle, this study sought to ascertain the effect of teaching derivatives in Vietnamese high schools. In this quasi-experimental study at a high school in Vietnam, there were 78 grade 11 students (40 in the experimental and 38 in the control classes). As opposed to the control class, which received traditional instruction, the experimental class's students were taught using the ACE learning cycle based on the APOS theory. The data was collected based on the pre-test, the post-test results and a survey of students' opinions. Also, the data that was gathered, both qualitatively and quantitatively, was examined using IBM SPSS Statistics (Version 26) predictive analytics software. The results showed that students in the experimental class who participated in learning activities based on the APOS theory improved their academic performance and attitudes. Additionally, it promoted the students' abilities to find solutions to problems about derivatives.

Keywords: *Academic achievement, ACE learning cycle, APOS theory, derivative, mathematics education.*

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Introduction

Educators must devise new teaching methods to resolve the tension between new-hire training and traditional instruction. In addition, the rapid expansion of today's knowledge-based economy makes it critically important to improve educational standards. In order to accomplish this objective, there is a strong emphasis placed on the process of conceptualizing and developing innovative educational strategies. Mathematics is a science that is useful in many contexts, including our daily lives and attempts to understand other sciences and technologies (Siswono et al., 2016). It can be learned in schools and has relevance to everyday life. Today, those who understand and are proficient in math have a better chance of shaping their future; each student should be given the opportunity and support to specialize in math. Mathematical learning theory explains students' needs and problems (Trigueros & Martinez-Planell, 2009). According to Dubinsky and McDonald (2001), any math teaching theory should explain how students learn math and suggest ways to improve the curriculum.

Piaget (1971) studied children's math and abstract thinking, and Dubinsky developed actions, processes, objects, and schemas (APOS) (Arnon et al., 2014). APOS theory has been applied to many high school and university math topics, including graphs of fractional functions (Tokgoz, 2015), improving understanding of graphs of functions and their derivatives (Borji, Font et al., 2018), linear equations (Rachmawati, 2020; Umam & Susandi, 2022), partial integrals (Borji

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& Font, 2019), line equations (Kamid et al., 2021). Besides, APOS theory applications help students better understand concepts, which improves mathematical communication and critical thinking (Marsitin, & Rahayu Sesanti, 2018).

The APOS theory and the activity, classroom discussion, and exercise (ACE) learning cycle are relatively new in mathematics education in Vietnam. Therefore, educators need to look into the characteristics, advantages, and difficulties of using the APOS theory and ACE learning cycle in mathematics education, particularly its efficiency and suitability for use in Vietnam. According to Nguyen et al. (2019), Vietnam's ongoing educational reform includes updating curricula and textbooks. Additionally, the APOS theory exemplified these learning characteristics to enhance mathematical education. The number of studies on the use of the APOS theory and ACE learning cycle in teaching and learning mathematics in the context of Vietnam's education system, however, is comparatively small, and no research has been done to determine how well the APOS theory and ACE learning cycle teach derivatives in Vietnam's high schools. Correspondingly, this study aims to ascertain whether teaching 11th-grade Vietnamese students in high schools about derivatives using the APOS theory and ACE learning cycle improves students' academic achievement, participation, motivation, and learning attitudes.

APOS Theory

APOS is a constructivist learning theory created by Dubinsky (2014). Finally, students need schemas to understand a new math concept better. Dubinsky (2002) identifies five abstract mental mechanisms: interiorization, coordination reversal, encapsulation, and de-encapsulation. These thinking mechanisms build actions, processes, objects, and schemas. The diagram below shows the APOS theory's intelligence structures and mechanisms.

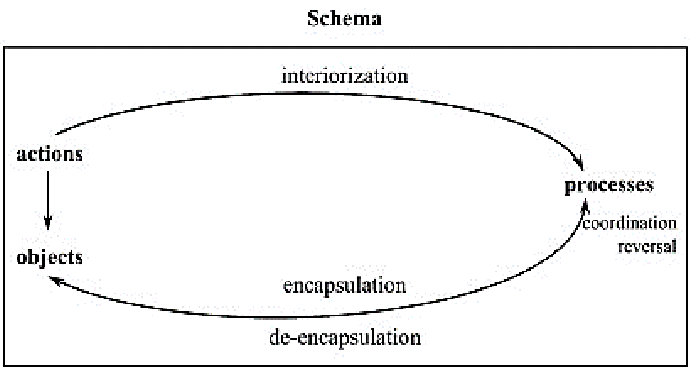

Figure 1. The Relationship Between Structure and Thinking Mechanism in the APOS Theory (Arnon et al., 2014)

Understanding a mathematical concept involves manipulating mental structures or real objects to form activities, which are then collected into processes and summarized to form new objects. Decapsulating new objects reveal their formation process. The schema is formed by reorganizing actions, processes, and objects (Asiala et al., 1996). According to Piaget (2006) and APOS theory, a concept is an action; action is an explicitly, externally guided transition of one or more previously conceived objects. Action leads to action (i.e., no step can be skipped). The action could be very easy or difficult to carry out, depending on the circumstances.

Dubinsky et al. (2005) found that repeating and thinking about an activity can turn it into a thought process. The process is a mental construct that performs the same action as codified but in the individual's mind, allowing them to perform transformational fantasy without doing each action. The number of steps needed to complete something can differentiate an action from a process. If a person perceives a process as a whole, and transformations can be applied to that whole and produce transformations (explicitly or in one's imagination), then the person has reduced the process to the object of perception (Dubinsky et al., 2005). The interaction of elements in Figure 1 creates schemas. As a result, a schema is built as a consistent set and connection of structures (Action, Process, Object, and Schema). It can be used as a static structure (object) or dynamic structure to assimilate other objects (or schema).

Applying APOS Theory to Analyzing Students' Mathematical Abilities

According to the APOS theory, a person's interest in solving a perceived problem through reflection on the issue and its resolution in a social context and by learning how to create or reconstruct actions, processes, and objects and organize them in schemas leads to the development of mathematical knowledge (Dubinsky, 2014). The terms action, process, object, and schema were first introduced by Cottrill et al. (1996) in a work where the authors used the APOS model to examine students' conceptual understanding of limits. Their study introduced the concept of genetic decomposition (the description of the mental structures that learners might go through when understanding a mathematical concept) to analyze the results obtained from the real world. Maharaj (2010, 2013) discussed ways to improve students' understanding of limits and derivatives using APOS theory. The APOS theory investigates students' thinking mechanisms and structures when they engage in demonstration activities to develop formal proof strategies (Syamsuri et al., 2017).

Using APOS and Objective Self-Awareness (OSA) theory, university students' understanding of a function and its derivative was tested. Most struggled to calculate derivatives at critical points and tangent slopes (Borji, Font et al., 2018). Each level of derivative development has features from the previous level and new levels to distinguish it, representing a conceptual advance (Fuentealba et al., 2018). University students struggle with complete proof (Syamsuri et al., 2017). Arnawa et al. (2007) proposed using APOS theory to teach Abstract Algebra because it improves students' proof ability. Herawaty et al. (2020) used the APOS theory to assess students' difficulties with closed-set properties.

Susandi et al. (2017) used APOS to analyze math pedagogical students' matrices mistakes. Trigueros and Martinez-Planell (2009) examined the relationship between students' conceptions of three-dimensional Cartesian space and their understanding of two-variable function graphs. Umam and Susandi (2022) used the Action-Process-Object-Schema (APOS) theory to identify students' errors in solving cases requiring the use of critical thinking skills from linear equations with two variables. The studies discussed earlier show that APOS is an influential theory for analyzing students' ability to comprehend concepts, so it can explain why some people master mathematical concepts while others struggle.

Applying APOS Theory to Teaching Mathematics

Applying APOS theory in math learning can improve students' understanding, Moll et al. (2015) said. The Modification of APOS (M-APOS) has improved students' mathematical reasoning ability (Nufus et al., 2018). One of the fundamental general skills that students need to develop early in their academic and personal lives is the ability to solve problems; a study by Istikomah and Jana (2019) shows that the M-APOS model is very effective for developing problem-solving capacity for students in the group theory course.

The ACE learning cycle is a teaching model used in research on applying APOS theory to education; it consists of three phases: activities (with computer software), classroom discussion, and exercises (Arnon et al., 2014). The ACE cycle has been used in elementary to university pedagogy. The APOS learning model improved students' logical and algorithmic thinking (Hartati, 2014). Salado and Trigueros (2015) used genetic decomposition to introduce feature values, eigenvectors, and vector spaces. According to research, using APOS theory to teach elementary linear algebra increases students' understanding (Arnawa et al., 2021). Also, APOS is a teaching theory that helps students master abstract algebra (Arnawa et al., 2007).

ACE Learning Cycle

APOS theory is applied to teaching mathematics through the ACE learning cycle (Arnon et al., 2014; Borji, Alamolhodaei et al., 2018), which is a pedagogical strategy embracing three components: (A) Activities; (C) Classroom discussion and (E) Exercises done outside of the classroom. The APOS theory requires an assumption to be made regarding a particular mathematical concept in order to organize learning activities properly. This finding is referred to as genetic decomposition, a term that describes the outcome of this analysis.

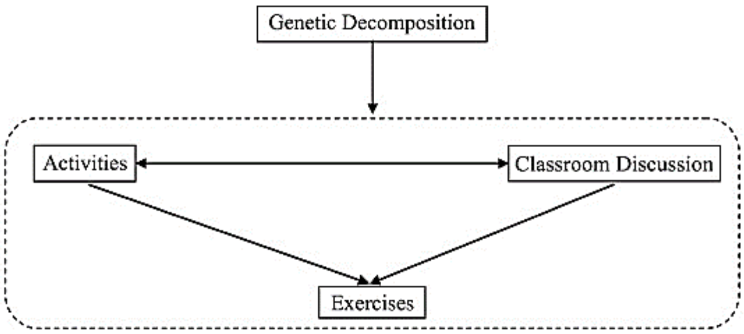

Figure 2. The Relationship Between Genetic Decomposition and the ACE Learning Cycle (Arnon et al., 2014)

According to Figure 2, the genetic decomposition of a mathematical concept is a sequence of mental structures to construct a mathematical concept that develops in one's mind. Thus, genetic decomposition requires a mental structure of actions, processes, objects, and schemas that describe certain mathematical concepts. The genetic decomposition's effect on the entirety of the ACE learning cycle is represented by the arrow that extends from it in this diagram. Activities are the main topic of classroom discussion, and discussion allows students to reflect on activities. The arrows from class activities and discussions to assignments show that the exercise's main purpose is to reinforce students' mental structures from class activities and discussions. Teachers should use IT and cooperative learning to complete operational phase learning tasks. The ACE learning cycle can be broken down into three distinct parts:

(1) Phase 1 (Activities): Students are asked to perform guided activities.

(2) Phase 2 (Classroom Discussion): Issues arising in phase 1 are discussed in a classroom setting, where classroom exercises are done with the teacher's help.

(3) Phase 3 (Exercises): Students are given problems to solve at home.

The cycle's activities will be in ascending order of difficulty and designed to adjust the student's mental structures. As they progress through the cycle, students must re-forge them into expected structures closer to the genetic decomposition. Lessons are taught in small groups with little instructor involvement. The class exercises will be harder than the previous ones but will reinforce the same mental structures. These exercises are difficult because they involve many geometric relationships, variables, and constants. The homework assignment summarizes the class assignment's most important points. Also, this problem is less exploratory than classroom activities and less complex. It is meant to help students organize and summarize their thoughts on the abovementioned issues.

Teaching Derivatives

The development of calculus is one of the greatest and most significant achievements of the human mind; its power and versatility can be seen in its ability to translate complex problems into simple rules and procedures in many fields, such as mathematics, physics, engineering, the social sciences, and biology (Berry & Nyman, 2003; Kleiner, 2001). Students can often solve problems that require algorithmic rules or procedures, but they struggle with problems related to understanding concepts (Selden et al., 1999). Calculus courses use a central concept called a derivative to study and understand varying magnitude phenomena. In calculus, derivatives play a pivotal role. Although not new, the problem of understanding derivatives remains one of the most significant challenges in higher education mathematics, a constant concern of higher education institutions because it leads to high failure rates and calculus course abandonment (Bressoud et al., 2015). Due to this, it is urgently necessary to find a teaching strategy that will enable students to understand derivatives and the knowledge that goes along with them.

Students struggle to connect the derivative graph to the original function (Orhun, 2012). The author proposed making derivatives and calculus more intuitive to improve their quality. Students often make conceptual, interpretive, linear extrapolation, procedural, and arbitrary errors when calculating exponential, logarithmic, and trigonometric derivatives (Siyepu, 2013). Most college students have trouble developing the thinking structures and performing the practical work needed to deal with the problem, especially the mental constructs needed to determine the derivative at the critical points and determine the rate of variation of the slope of the tangent (Borji, Font et al., 2018). Zengin (2018) used GeoGebra and the ACODESA method to study how pedagogy students relate derivative and differential to integral. Students often think one variable can have two symbols, which is wrong. Nurwahyu et al. (2020) showed that students' derivatives directly relate to their reasoning ability by citing common errors in solving math problems and misconceptions in applying basic math formulas. Also, Sahin et al. (2015) conducted a study using modeling to deepen their comprehension of the concept of the derivative.

A study by two authors, Lan and Zhou (2020), used a six-question cognitive model to explore the concept of derivative from six aspects: "from where", "what", "why", "how", "what if it changed," and "think about it". Similarly, Park (2015) discusses how derivatives should be taught. Research shows that this model helps students understand derivatives. To help students understand limits, Baye et al. (2021) used GeoGebra software and flexible teaching methods based on APOS theory. Both qualitative and quantitative data show that learning improves students' understanding of limits. As a result of this finding, instruction in mathematics centered around technology use is an excellent choice.

When learning derivatives and integrals, most challenges students face are related to problem-solving. This is especially true for learners with poor writing and speaking skills (Hashemi et al., 2015). In their study, the authors present a learning strategy based on mathematical thinking and generalization strategies, with prompts and questions. Understanding derivatives are key for economists (Feudel & Biehler, 2020). The study uncovered several misunderstandings and knowledge gaps among students about derivatives in economics.

Each piece of literature offers a viewpoint or mode of instruction that simplifies learning by focusing on derivatives. The number of studies on applying APOS theory to math classes in Vietnam is very limited. In particular, there has not been any research on instructing derivatives at high schools associated with this theory. Analyzing recently published derivative works has shown that students struggle with this concept. When teaching derivatives using APOS theory, it is assumed that most students will have trouble understanding and using the concept directly (Arnon et al., 2014).

Teaching Derivatives in High Schools in Vietnam

Derivatives are essential in math and physics for finding tangent slopes and instantaneous velocities. Specifically, Table 1 lists the fundamental knowledge students in high schools across Vietnam are expected to have regarding derivatives.

Table 1. Required Knowledge and Skills in Derivatives in High Schools

Derivative	**Level to be achieved**	
	Knowledge	**Skills**
Concept of derivative	Know the definition of the derivative (at a point, over an interval). Know the mechanical and geometrical meaning of derivatives	By definition, calculate the derivative of a power function, a polynomial function of the second or third order. Write an equation for a tangent to a function's graph at a given point on the graph. Know how to find the instantaneous velocity with the equation at one instant of a motion $S = f(t)$.
Rules for calculating derivatives	Know the rules for calculating the derivative of the sum, difference, product, and quotient of functions, composite function and derivative of a composite function.	Calculate the derivative of the function given in the above forms.
Derivatives of trigonometric functions	Know the limit (no proof required) $\lim_{x\to 0}\frac{\sin x}{x} = 1$. Know the derivative of a trigonometric function.	Calculate the derivative of some trigonometric functions.
The second derivative	Know the definition of the second derivative.	Calculate the second derivative of some functions. Calculate the instantaneous acceleration of a motion with the equation $S = f(t)$ given.

The concepts of derivatives are often seen as too abstract and difficult for high school students to understand. Applying APOS theory to teaching derivatives will improve math and calculus instruction. As a result, they will help students' math skills. The topic of implementing APOS theory into Vietnam's math instruction has not been adequately addressed. In addition, the application of APOS theory to the process of teaching and learning is founded on assumptions, including an important assumption regarding learning: Students do not absorb mathematical concepts directly; rather, students manipulate structures in order to make meaning of mathematical concepts. High school students find derivatives too abstract and complex to study directly. Education aims to equip students to address everyday issues, overcome challenges, and identify the best responses to practice-based issues.

This research aimed to examine the effectiveness of introducing derivatives to Vietnamese high school students via the APOS theory and the ACE learning cycle. The research questions are pertinent to the previously stated research goal:

1. Can students learn more efficiently and get better results due to the APOS theory and ACE learning cycle?

2. Is there a discernible difference between students taught using the APOS theory and ACE learning cycle and those instructed using the conventional approach regarding academic achievement?

3. How have the APOS theory and ACE learning cycle affected students' participation, motivation, and attitude toward mathematics learning?

Methodology

Research Design

The experiment was conducted from March 21, 2022, to April 2, 2022, and was conducted on 78 students in grades 11A3 and 11A7 of Luu Huu Phuoc High school, O Mon District, Can Tho City, Vietnam. Students and their parents were informed of their participation in the experiment. Furthermore, the research did not uncover any unfavorable repercussions for the students. The experimental class consisted of students who would learn the topic of derivatives (Textbook of Algebra and Calculus 11) by the ACE learning cycle, while the control class would be taught according to the conventional teaching model. Thus, traditional didactic lectures were given to participants in the control group. In other words, they did not gain anything from the learning process about the ACE learning cycle and APOS theory compared to the experimental class. Besides, the topic that would be covered was not disclosed to this group. The students in the control group were neither encouraged nor discouraged from asking questions throughout the course, and the lecture was not broken up into smaller subtopics. Additionally, the research team organized and led a training seminar for ten teachers to assist them in attaining the professional qualifications necessary to instruct learning scenarios on derivatives, which were constructed according to the APOS theory and ACE learning cycle. From there, a teacher was selected because he demonstrated proficiency in implementing the APOS theory and ACE learning cycle fundamentals. Also, he had over ten years of experience teaching mathematics in high schools.

This study considered school-created groups rather than a random sample, so the method was quasi-experimental. Indeed, a quasi-experimental was conducted to determine the effect of the instructional treatment on that basis to test the research hypothesis; this method was similar to the study of Baye et al. (2021) and Borji, Alamolhodaei et al. (2018).

Also, the design in this study included the methods used by Gravetter and Forzano (2018). The learning approach was an independent variable in this study because it was measured independently (APOS theory, ACE learning cycle and conventional instruction). Besides, the student's academic achievement level served as the dependent variable in this experiment. The procedure for data collection is outlined in Table 2, which can be accessed here. This design, as below, was also done similarly in the studies of Aminah et al. (2018) and Winarti et al. (2022).

Table 2. Design of a Quasi-Experimental Study

Group	Pre-test	Treatment	Post-test
The experimental class	Regular test scores at the end of the chapter on limits	Lessons plans with learning tasks designed according to the ACE learning cycle	Post-test Student opinion survey
The control class	Regular test scores at the end of the chapter on limits	Lessons in the traditional, lecture-based way	Post-test

Analyzing of Data

While a control class was given the pre-test to complete, the researchers created pre-and post-tests to give to an experimental group. Quantitative analysis was done on the data from the pre-and post-tests. IBM SPSS Statistics (Version 26) predictive analytics software added a quantitative analysis to the t-test to examine how the experimental and control groups' mean values differed. The research team was tasked with conducting an experimental process related to the viability and effectiveness of the teaching process and with teaching, observing, and gathering data reflecting that process. Regarding the validity and reliability of instruments, the experimental lesson plans were moderated by experts in mathematical education at Can Tho University, and the tests were verified by colleagues who were teachers. Teachers at the school conducted experiments to ensure the lesson objectives were specified in the program. Upon completion of the adjustments recommended by the experts, the tools were found to be appropriate in terms of academic content and ability to assess student academic achievement and may be used when experimenting. The results of the experiments performed throughout the study were evaluated using both qualitative and quantitative methods. The Shapiro-Wilk test tested both groups' pre-and post-test scores data for normal distribution. An independent t-test (2-tailed) was used to compare the mean of two experimental and control classes, while paired (2-tailed) t-test was used to compare the mean between two sets of scores before and after the effect of the experimental class. To test the correlation coefficient Pearson, the degree of influence was equal to using Cohen et al. (2017) mean deviation formula.

Specifically, a set of 8 survey questions was designed according to a 5-level Likert scale to assess students' attitudes towards the ACE teaching method in the experimental class and learning. It was expected that students could self-assess their problem-solving abilities. A total of four items on a Likert scale with five levels are included in the student survey statements to assess attitudes: Strongly disagree - disagree - neutral - agree - strongly agree. One item (4) is a multiple-choice statement about the student's favorite learning activities. The Google Forms program was used to create and distribute the survey, which was mandatory for the experimental group's students to complete. Besides, a Cronbach's alpha test was performed to check the reliability of the experimental class student attitude scale and obtained the following results: Observed variables (8 observation questions) have Cronbach's alpha coefficient of 0.88 (large) more than 0.7), and the variable-total correlation coefficient (corrected item-total correlation) was suitable (not less than 0.3). For this reason, the scale met the requirements of internal reliability. As supporting evidence for the research questions, data from pre-and post-tests, worksheets, observations, and survey results were analyzed quantitatively and qualitatively.

Design of Experiment

The planning and organization of the research went through the two phases outlined below.

Phase 1: Genetic decomposition of the topics of derivatives

Connecting two points $A\big(a, f(a)\big)$ and $B\big(b, f(b)\big)$ on a curve $y = f(x)$ to construct the corresponding line of the curve and the operations required to calculate the slope (inclination) $m = \frac{f(b)-f(a)}{b-a}$ on that line.

- The above operations are absorbed into calculating the slopes of the lines at the point, and point b gets close to the point (the sand line becomes the tangent).
- Summarizing the above process into two objects, the tangent at the point $\big(a, f(a)\big)$on the graph and the tangent's slope, i.e., the derivative $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ of the function $f(x)$ at a point $x = a$.
- When the above actions, processes, and objects are closely organized in a diagram, students can resolve problems related to a function's derivatives (especially graphs).

i) Definition of the composite function

- Identifying the function $f[u(x)]$ when knowing two functions $y = f(u)$ and $u = u(x)$.
- Acquiring the above activity into the determination process of the function $f[u(x)]$ and reversing the above process to determine the function $y = f(u)$ and intermediate function $u = u(x)$ from the function $f[u(x)]$.
- Summarizing the above processes into a new object is a composite function.

ii) Derivative of the composite function

- Calculating the value of the derivative at a point of some familiar functions $f(x) = x^n, g(x) = \sqrt{x}$. Using a pocket calculator, calculate the derivative at a point of composite functions $f(x), g(x)$made up of any intermediate composite function $u(x)$.
- Incorporating the above operations into a procedure for calculating the derivative of a composite function at a given point.
- Summarizing the above process into a theorem of theorem 4 (Hao et al., 2015)
- When the above actions, processes and objects are closely organized in a schema, students can solve problems related to derivatives of a composite function made from regular functions.

Phase 2: The practice of teaching the topics of derivatives

During the research project, the research team and the teacher discussed designing lesson plans based on the above genetic decomposition results; the learning activities in these lessons assigned to the experimental class were designed based on the ACE learning cycle, except for the control class, another teacher conducted teaching according to the conventional method. Finally, the authors gave the experimental and control classes a post-test, analyzing the classroom atmosphere in the two groups and the students' opinions in the experimental class to check the student participation, motivation and learning attitudes.

Lesson plans designed and instructed included teaching the concept of derivative (contains contents about equations of the tangent of a given function at a point, a theorem on the relationship between continuity and derivative; derivative of some ordinary functions meet; derivative of the sum, difference, product, quotient) and teaching the derivative of a composite function and its derivative. The following section illustrates the activities based on the ACE learning cycle included in the first lesson on derivatives.

Illustrating a situation of teaching the concept of derivatives and tangent equations of a given function at a point using the ACE learning cycle.

Activities (Supported by GeoGebra Software)

Learning task 1: Determine the slope (slope, inclination) of the following lines:

a) $f(x) = 2x + 1$

b) $f(x) = -2x$

c) $f(x) = 1$

This activity helps students review how to determine the slope of a straight line $y = ax + b$ and write a line when the slope is known (for applying the derivative of a function to write a tangent at a given point).

Learning task 2: Draw the graph of the function $f(x) = x^2$. Draw and determine the slope of the sand line passing through $(a, f(a))$ and $(b, f(b))$ in turn for the following cases:

a) $a = 1, b = 2$

b) $a = 1, b = 1.2$

c) $a = 1, b = 1.01$

Comments: For drawing the graph of the function $f(x) = x^2$, students can easily complete it, but for tasks related to determining the slope of the linear slope, it will not be easy, so teachers should use GeoGebra software to support students. Thus, Figure 3 provides a visual representation of the calculation for determining the slope of the secant.

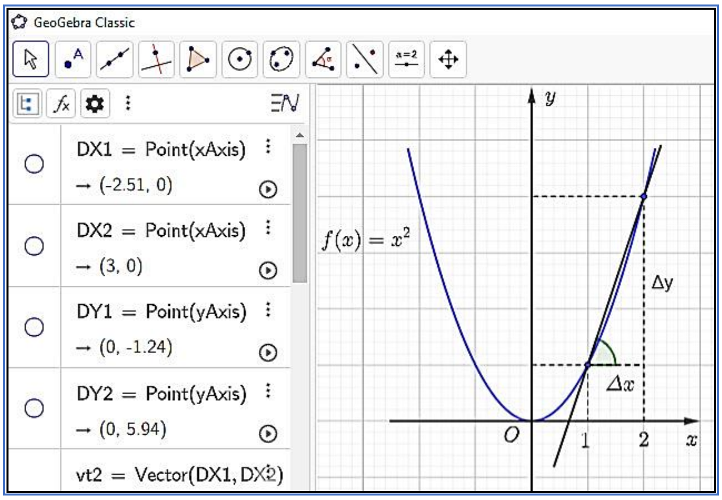

Figure 3. The Geometrical Meaning of the Derivative

Also, Figure 3 illustrates the application of GeoGebra software (the first requirement of learning task 2), the mistakenly used software helps students have the most accurate view of the graph of the function $f(x) = x^2$ and slope of the secant passing through two given points on the graph.

Classroom Discussion

Students are tasked with understanding a function's derivative at a particular point by beginning with the definition of a tangent to a curve.

Learning task 3: If two points $(a, f(a))$, $(b, f(b))$get closer and closer together (to the extent $a \approx b \approx 1$), what is the slope of the line? Hint: Use knowledge of limits to calculate this slope.

Indeed, the relationship between the tangent and the function's graph is illustrated in Figure 4.

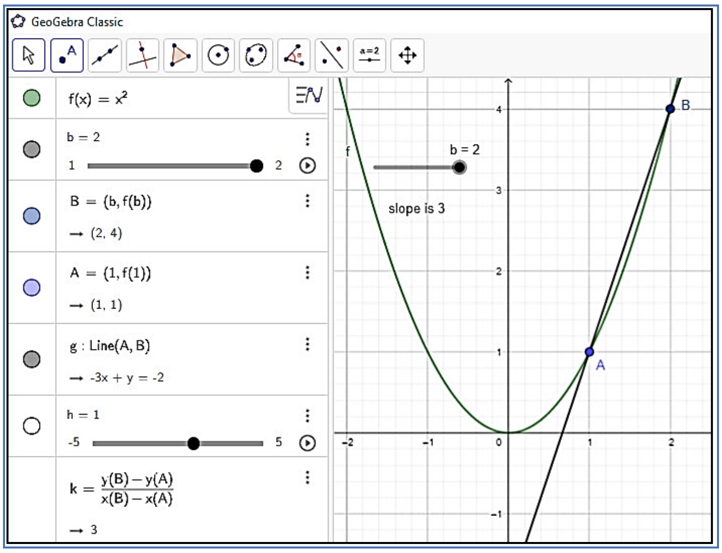

Figure 4. The Geometrical Meaning of the Derivative

The use of GeoGebra software to assist students in observing the secant's shape and the slope is demonstrated in Figure 4 (the student can change the variable value b by using the slider); this line will have a tangent form when $a \approx b \approx 1$.

Comments: When students can give discrete calculations in task 2 in terms of the form $\lim_{b \to a} \frac{f(b) - f(a)}{b - a}$ and calculate the result by 2, students have a basic grasp of the concept of the derivative of a function at a point. At this time, the teacher introduces the concept of derivative to students and aims to calculate the function's derivative $f(x) = x^2$ at other points.

Learning task 4: State the procedure to determine the derivative of a function at a given point, then apply this procedure to calculate the derivative of the function $f(x) = x^2$ at the point $(x_0, f(x_0))$ corresponding to each of the following cases:

a) $x_0 = -1$

b) $x_0 = 1$

c) $x_0 = 0$

Learning task 5: Put $\Delta x = x - x_0$, and $\Delta y = f(x + \Delta x) - f(x)$, then the procedure of finding the function's derivative at a given point becomes the finding of the limit $\lim_{\Delta x \to 0} \frac{\Delta x}{\Delta y}$. Students are asked to apply this rule and find the derivative of the function $f(x) = x^2$ corresponding to each case in the learning task 4.

Learning task 6: Using the derivative at a given point as the slope, write the general form of the equation of the tangent to the function $y = f(x)$, knowing that the tangent passes through the point $(x_0, f(x_0))$.

Learning task 7: Write the equation of the tangent to the function graph $f(x) = x^2$ for each point in learning task 4.

Exercises

Learning task 8: Calculate the derivative of each of the following functions (by definition) and write an equation for the tangent at the indicated points:

a) $y = x^2 + x$ tại $x_0 = 1$

b) $y = \frac{1}{x}$ tại $x_0 = 2$

c) $y = \frac{x+1}{x-1}$ tại $x_0 = 0$

Comments: By employing this exercise, students are guided through a review of the knowledge system of derivative ideas, including computation, the meaning of the concept, and how to apply it to problem-solving.

Results

Results Relating to the Pre-test

Before making the instructional intervention, a pre-test was needed to compare the experimental and control groups' knowledge levels. This was done so the educational intervention's success could be determined. The following table shows the experimental and control groups' pre-intervention scores.

Table 3. Independent Samples t-Test and Assumption Checks (Pre-test)

Group Statistics				
Group	**N**	**Mean**	**Std. Deviation**	**Std. Error Mean**
Experimental goup	40	6.381	1.416	0.224
Control group	38	6.232	1.885	0.306
Levene's Test for Equality of Variances				
F		Sig.		
3.662		.059		
t-Test for Equality of Means				
	t	**df**	**Sig. (2-tailed)**	**Mean Difference**
Equal variances assumed	0.398	76	.692	0.150

Table 3 shows that experimental and control groups had similar pre-test scores, 6.381 and 6.232. A t-test is used to test the hypothesis of mean equality. Before running the t-test, we compare the two classes' pre-test scores using the Levene test. Table 3 shows a Levene test with a significance level (Sig.) of .059, indicating no variance between the two groups; experimental class scores are equivalent to control class scores. A t-test with two-class scores and equal variance was performed. The observed significance level (Sig.) is .692, greater than .05, showing no difference in mean score value between the experimental and control classes. The above research results show that experimental and control groups have similar academic achievements.

Results Relating to Post-test

A 5-question post-test was designed to test mathematical problem-solving skills and serve as a basis for quantitative analysis of treatment effects. Particularly, the test's content was developed based on the derivatives-related criteria that had to be satisfied.

Table 4. Independent Samples t-Test and Assumption Checks (Post-test)

Group Statistics					
Group	**N**	**Mean**		**Std. Deviation**	**Std. Error Mean**
Experimental goup	40	7.038		1.254	0.198
Control group	38	6.369		1.244	0.202
Levene's Test for Equality of Variances					
F			**Sig.**		
0.159			0.691		
t-test for Equality of Means					
		t	**df**	**Sig. (2-tailed)**	**Mean Difference**
Equal variances assumed		2.365	76	0.021	0.669

The results of the experimental and control classes are presented in Table 4, where the mean scores are shown to be 7.038 and 6.369, respectively. As a result, the experimental group's mean score is higher than the control group's. Moreover, an independent t-test is used to compare the two means. The results of the t-test are summarized in the table that can be found below.

A Levene test compares the two classes' post-test variances. The Levene test results show no difference in the variance of the two classes of scores because the observed significance level was .691, greater than .05. Since then, a t-test has been run if there is no difference in score variance between experimental and control classes. The significance level is .021, less than .05. That means the mean score difference between experimental and control classes is significant; test results show a difference in academic performance.

Moreover, Table 4 shows that the experimental class scored higher than the control class. The independent t-test shows the significance of the two classes' mean scores. In conclusion, the experimental class outperformed the control class. The experimental class had better problem-solving skills than the control class, which was discovered indirectly. This study investigates the possibility that the differences in outcomes are not caused by the ACE learning cycle but rather by other factors (such as the caliber of the subject or random factors). A pair sample t-test was conducted (still with a significance level of 0.05) on the scores before and after the experimental class to demonstrate that the experimental class's higher score directly results from the ACE learning method. This demonstrated that the implementation of ACE contributed to the higher score achieved by the experimental class. The results are presented down below.

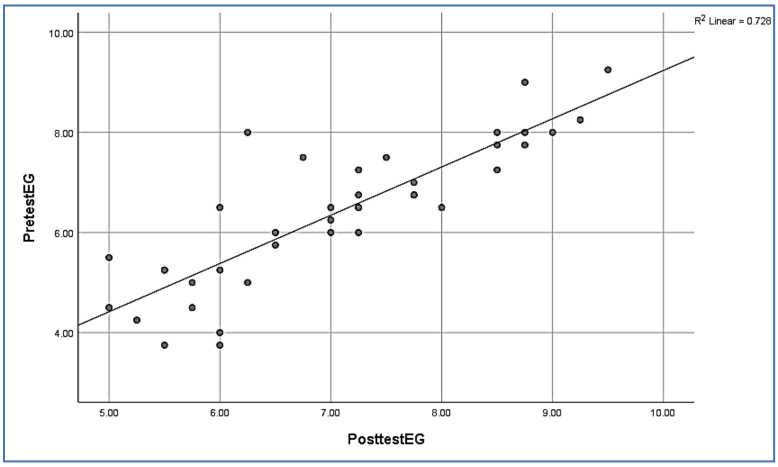

Figure 5. Scatter Chart of Test Scores before and After the Treatment of the Experimental Class

The graph in Figure 5 that can be seen above depicts most scores clustered around a straight line (showing a positive linear relationship). It appears that those students in the experimental class who had high results on the test before this one will also have high results after it. In other words, the results of two tests performed before and after the treatment exhibited a correlation. In order to verify the correlation described above, a correlation test is carried out. Additionally, this test looks at the correlation's statistical significance and the Pearson correlation coefficient.

Table 5. Pair Samples t-Test and Correlation

		N	**Correlation**	**Mean**	**Sig. (2-tailed)**
Pair	PRE&POS	40	.853	-0.656	.000

The significance level (Sig.) of the correlation test was 0.000, which is less than 0.01, so it can be confirmed that the correlation of scores before and after the intervention is statistically significant. Meanwhile, the Pearson correlation coefficient of the test results for the experimental class before and after the instructional intervention is 0.853, as shown in Table 5. According to the Hopkins table provided as a reference, this is a very high correlation. The conclusion that the correlation comment derived from the scatter plot analysis is correct is supported by the fact that this is the case. The last step was to conduct a paired t-test to determine whether or not there was a statistically significant difference between the test scores of the experimental class and those of the control group. The average score difference before and after the impact of students in the experimental class is statistically significant, as shown by the value of the two-sided observed significance level, which is .000 less than .005. Specifically, the average difference between scores before and after the treatment, which was -0.656, and the fact that this difference was negative allows one to conclude that the ACE learning cycle has positively impacted students' learning achievement.

The formula for calculating the influence (standard mean deviation) developed by Cohen et al. (2017) is applied to carry out the evaluation that determines how much of an impact the ACE learning cycle had on the overall academic performance of the enrolled students in the experimental class. Thus, the value of 0.54 calculated using this formula falls within the interval of values ranging from 0.5 to 0.79, indicating that the instructional effect was relatively moderate. Consequently, the ACE learning cycle reasonably impacted students and made learning more efficient. After receiving the instructional treatment, it is clear from the initial analysis that the performance of the experimental class is superior to that of the control class. Even though the instructional treatment did not impact the performance of the control class, this is still the case. After the experimental and control classes completed the experiments, the spectral differentiation of the student's test scores is presented in the following table.

Table 6. The Results of the Post-test

Classification	**Poor**	**Weak**	**Medium**	**Good**	**Very good**
Point range	0; 3.5)	3.5; 5)	5; 6.5)	6.5; 8)	[8; 10]
The experimental class	0	0	14	16	10
	0%	0%	35%	40%	25%
The control class	0	5	12	17	4
	0%	13%	31.7%	44.8%	10.5%

According to the statistical findings in Table 6, all forty students in the experimental group earned a grade of average or better, whereas only five of the forty students in the control group managed to earn an average grade. Especially the experimental and control groups almost had the same proportion of students with average and good grades in their respective classes (75 percent of students in the experimental class and 76.5 percent in the control class). Specifically, the number of students who achieved good scores in the experimental class was twice as high as those who did so in the control class, with a percentage of students who received good grades that were relatively high (25 percent; 10.5 percent). Based on lessons, the experimental class's learning outcomes in derivatives were considered superior to those of the control class (Lessons 1 and 2).

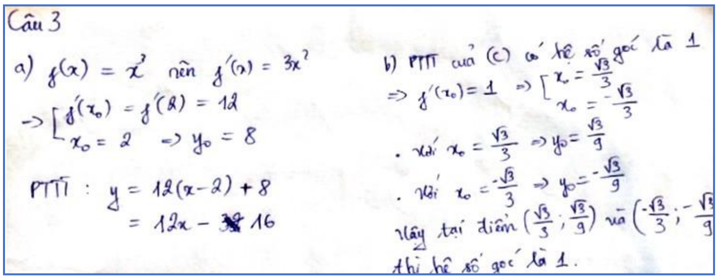

Figure 6. A Work of the Experimental Class Student (Good Work)

The work displayed in Figure 6 demonstrates that the student in question comprehended and competently applied the geometrical meaning of the derivative in order to calculate the slope and compose the equation of the tangent at a specific point. According to the APOS theory, the student did a fantastic job constructing a mental structure at the schema level.

Student Opinion Survey Results

After completing the practical lessons, 40 students of class 11A3 (experimental class) participated in giving opinions about the lessons through a survey. Specifically, the survey consisted of 8 questions to collect students' opinions on the effectiveness and level of interest in lessons and for students to self-assess their problem-solving ability. The questions in the survey were built on a 5-point Likert scale (strongly disagree, disagree, neutral, agree and strongly disagree).

Table 7. Statistics of Students' Opinions

Question	Statistics	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1	Frequency	0	1	9	11	19
	%	0	2.5	22.5	27.5	47.5
2	Frequency	1	1	11	14	13
	%	2.5	2.5	27.5	35	32.5
3	Frequency	2	1	8	11	18
	%	5	2.5	20	27.5	45
4	Frequency	1	1	11	16	11
	%	2.5	2.5	27.5	40	27.5
5	Frequency	0	1	13	14	12
	%	0	2.5	32.5	35	30
6	Frequency	1	1	10	14	14
	%	2.5	2.5	25	35	35
7	Frequency	0	2	9	16	13
	%	0	5	22.5	40	32.5
8	Frequency	2	1	8	16	13
	%	5	2.5	20	40	32.5

Question 1. What level of interest do you have in learning about derivatives?

According to the findings presented in Table 7, the vast majority of students in the experimental class had a favorable attitude toward the lessons that were developed by the ACE learning cycle (which accounted for 75 percent of the class), and only one student (a ratio of 2.5 percent) was not satisfied with these lessons. As a result, most students in the experimental class were excited about gaining knowledge about this subject through the ACE learning cycle.

Question 2. I feel that this learning process helps me better understand difficult and abstract concepts.

Students were encouraged to evaluate how well they understood the content discussed in the derivatives section and how that content related to the context by responding to the question. According to Table 7, most students were pleased with this method of instruction (accounting for 67.5 percent). Especially two students, or five percent of the class, perceived the educational material as ambiguous and unclear. As a result of classroom observation, it was possible to ascertain that the initial cause was the group discussion in a relatively short time; the responsibility for completing the task was not distributed among the group members. Overall, this could still be considered a meaningful response to the study, which showed that students had a relatively high level of comprehension of challenging concepts when instructed to study using the ACE learning cycle.

Question 3. The activities to start the lesson with the help of a pocket computer, GeoGebra software, etc., helped me to be more interested.

According to Table 7, most students in the experimental class reported that the warm-up activities to enter the lesson with the help of pocket computers, GeoGebra software, etc., helped them to be more interested. Since there were 29 students in this class, this result demonstrated that students learned new information more actively. Nevertheless, three students, representing 7.5 percent of the class, continued to disagree with the design of the warm-up activities; this was also a suggestion for the design of activities to be more straightforward and interesting.

Question 4. Group discussion activities in class help me understand the lesson and knowledge system better.

The goal of the question was to get students to reflect on the content covered in the ACE learning cycle's second phase. According to Table 7, most students enjoyed participating in activities related to class discussions (27 students). Besides, the responses obtained from this question were mostly favorable, indicating that the built-in group activities assisted the children in enhancing their understanding of the mathematical material about the topic of derivation, thereby contributing significantly to the growth of student academic achievement.

Question 5. I find that the activities of generalization and proof help me practice my ability to analyze and synthesize relevant knowledge and help me better understand concepts and the relationship between concepts.

Table 7 showed that a relatively high percentage of students agreed (55 percent) that proof and generalization helped them better understand concepts and their relationships. Only one student (2.5 percent) disagreed with this statement. The fact that this demonstrated that the demonstration and generalization activities incorporated into the designed discussions were quite suitable was a prerequisite to developing the ability to resolve problems.

Question 6. Consolidation activities (Exercises) help me to systematize what I have learned effectively.

The findings presented in Table 7 demonstrated that 28 students, or 70 percent, agreed and completely agreed about the effectiveness of the designed homework; it assisted them in more effectively organizing the information they already knew. Despite this, two students struggled with their assigned homework or five percent.

Question 7. I find myself making progress in solving problems related to the concept of derivatives.

The students received an orientation as a question to guide them through the self-evaluation procedure. According to the findings in Table 7, most students successfully solved problems relating to derivatives (72.5%), and only two students (5% of the total) did not notice that they improved their ability to solve problems.

Question 8. I would like to take similar classes on other topics.

The findings in Table 7 demonstrated that students valued the practical lessons developed by the ACE learning cycle. This was demonstrated by 29 students, accounting for 72.5 percent of the total, who desired to take the same lessons despite being focused on different subjects. Despite this, three students, representing 7.5 percent of the total, still did not want to take the same classes. Nevertheless, even though the experimental class's post-test analytical results were higher than those of the control class, this issue began in the first place because this was a relatively challenging topic that included a lot of abstract ideas. Besides, a few students in the experimental class had difficulty understanding the material.

Discussion

The effectiveness and viability of applying APOS theory to teaching derivatives in high schools were evaluated using qualitative and quantitative analyses of experimental data, such as before and after scores, classroom observations, and student opinion surveys. Indeed, post-test results showed that experimental and control class students met the course's knowledge requirements. The experimental group had an overall average score that was higher than that of the control group. Research by the authors Arnawa et al. (2021), Istikomah and Jana (2019) and Maharaj (2013) also tested that student average scores were enhanced after learning with the APOS theory. Besides, these results match those of Hartati (2014), Salado and Trigueros (2015), and Arnawa et al. (2021). A correlation test confirmed that the experimental class's higher scores were due to ACE's efficiency (and not other random factors). According to the Hopkins reference table, the experimental group's pre-and post-test scores had a high correlation (0.853). It can be said that students in the experimental group with high scores before the intervention also achieved high results in the post-test. Nonetheless, Figure 5 shows that the pre-test can account for 72.8% of the changes in the post-test. This indicates that other factors can account for only 28.2% of the variance. It can be hypothesized that some of the students in the experimental class had scores independent of the intervention in this context.

The practical lessons showed that students were more involved and active in learning and had more opportunities to practice skills in class. Most students in the experimental class were inquisitive and curious, and the warm-up problem stuck in their minds due to GeoGebra dynamic software (Baye et al., 2021). Motivating students to take the initiative, exercise their creativity, and exhibit a positive attitude toward acquiring new knowledge by having them work together in small groups to generalize and condense their previous mathematical conversations into a new mathematical object. Hence, they improved their ability to work together as a group as a direct result of the discussion.

Experimental class survey responses showed interest in ACE cycle lessons (accounting for 75 percent). In question 7, students were asked to rate how well they were able to solve mathematical problems. It was observed that most students had seen improvements in their problem-solving abilities (the agreement rate was 72.5 percent). The application of APOS theory and the ACE learning cycle to learning to promote students' problem-solving capacity is also mentioned in the research of Istikomah and Jana (2019) and Rahayu et al. (2023). Additionally, they desired to expand their knowledge through the ACE learning cycle (72.5 percent). The findings were similar to those of Hashemi et al. (2015).

Experiments have investigated the practicability and efficiency of ACE-based educational settings. Most control class students had trouble understanding the derivative and its geometric meaning, making it difficult to solve conceptual problems like expressing the tangent to a curve. Siyepu (2013), Borji, Font et al. (2018), Zengin (2018), and Nurwahyu et al. (2020) found similar results. The ACE cycle helped students in the experimental class understand and apply derivatives to mathematical problems, similar to Arnawa et al. (2021). For the experimental class taught by the ACE cycle with a computer (typically GeoGebra), the "concept image" of the derivative was clarified, which improved their understanding (Nurwahyu et al., 2020; Sahin et al., 2015).

Conclusion

This study sought to determine how well derivatives are introduced to Vietnamese high school students through the APOS theory and the ACE learning cycle. The experiment's findings provided sufficient data to accomplish the research purpose and answer the research questions. Indeed, applying the APOS theory and the ACE learning cycle to instructing derivatives helped students learn more effectively and get better academic performances, developed students' problem-solving skills (especially tangent problems), and actively and enthusiastically participated in activities to form new knowledge. Salado and Trigueros (2015), Moll et al. (2015), Nufus et al. (2018), Borji, Alamolhodaei et al. (2018), Baye et al. (2021), and Arnawa et al. (2021) found similar results. One of the main contributions of the current study to mathematics education literature is using the GeoGebra software, based on the ACE cycle, to teach and promote students' understanding of derivative topics.

In conclusion, the APOS theory and the ACE learning cycle were applied in this study to examine students' understanding of derivatives. Although there is much research on students' understanding of derivatives in Calculus education, there is very little work on teaching and learning these topics in conjunction with GeoGebra software. The use of APOS, ACE, and GeoGebra together improved our understanding of how well students understood derivatives, and the networking of these theories and software can assist researchers in examining how well students comprehend other mathematical concepts and related knowledge.

Recommendations

Experimental results show that teaching derivatives using the ACE cycle and GeoGebra software improves students' understanding and problem-solving ability. This paves the way for additional research into how APOS theory and GeoGebra can improve education across many subject areas. Moreover, the goal of future study is to examine how derivatives are taught in calculus lectures, both conceptually and procedurally, and how each method of instruction affects students' comprehension. Future research involving a larger sample size or population is advised to provide consistency and stability to the findings of this study.

Based on the derivative's problem-solving skills, the APOS theory and ACE learning cycle may apply to teachers and lecturers instructing this subject. The fact that the derivative can solve problems lends credence to this theory. Besides, more research is needed to design appropriate instructional activities using the APOS-ACE framework and technology (such as GeoGebra) to improve students' mathematical competencies.

Although the method's effect size was modest, it is a positive indication of the beneficial effects of the ACE learning cycle on students' math achievement. The software can teach students statistical probability, spatial geometry, and integrals because using APOS theory to teach math is simple. Thus, this plan will benefit calculus education both generally and specifically. Also, teaching models that encourage real-world application are required because derivatives appear connected to real-life circumstances. This presentation aims to introduce the use of RME in the Vietnamese mathematics education system (Nguyen et al., 2020).

Limitations

Nevertheless, the study does have a few drawbacks to consider. The experimental time was quite short, so the experiment had not clearly shown the development of the problem-solving ability of some students in the experimental class. Additionally, because the time allotted for the class discussion activities was restricted, not all students could accomplish the desired outcomes.

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Authorship Contribution Statement

Nga: Concept and design, technical or material support. Dung: Editing and writing. Trung: Supervision, admin. Nguyen: Critical revision of the manuscript, final approval. Tong: Writing, editing/review. Van: Data acquisition, data analysis, statistical analysis. Uyen: Drafting manuscript, supervision.

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