Research Article doi: 10.12973/eu-jer.9.1.129

European Journal of Educational Research
Volume 9, Issue 1, 129 - 141.
ISSN: 2165-8714
http://www.eu-jer.com/

Investigating the Development of Pre-Service Teachers' Problem-Solving Strategies via Problem-Solving Mathematics Classes

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Received: October 24, 2019 ▪ Revised: December 16, 2019 ▪ Accepted: December 20, 2019

Abstract: This study investigated the development of problem-solving strategies demonstrated by 42 elementary pre-service mathematics teachers in problem-solving mathematics classes. The study used a mixed methods approach of quantitative and qualitative research by analyzing the collected data.. The quantitative portion calculates the frequencies and percentage of the participants' responses to the problems posed in three different phases of the intervention: before, during, and after receiving the mathematics lessons. The qualitative approach was used for in-depth investigation to describe various mathematical problem-solving strategies demonstrated by participants across the three different research phases. Findings of the study indicated a limited number of problem-solving strategies present during the first phase of research such as "use arithmetic operation strategy," and "make a drawing strategy,". During the implementation of the problem-solving lessons and classroom discussion, the participants began to develop more strategies such as "use logical reasoning," "solve a simpler problem," "guess and check," "organize data in a table or a list," "look for a pattern," "work backwards," and "solve an equation,". However, the research findings nonetheless revealed participants' weakness in applying the variety of skills required for success in problem solving, such as interpreting information, mathematical working, and logical thinking. Results also demonstrated a limited and incorrect use of mathematical terminology, as well as a lack of problem comprehension. The discussion of the study addresses different features and issues related to mathematical problem-solving strategies. In light of its findings, this study presents recommendations and suggestions for the future development of pre-service teachers' problem-solving strategies.

Keywords: *Mathematics problem solving, problem-solving strategies, pre-service mathematics teachers, development of problem-solving strategies.*

To cite this article: Barham, A. I. (2020). Investigating the development of pre-service teachers' problem-solving strategies via problem-solving mathematics classes. *European Journal of Educational Research*, *9*(1), 129-141. https://doi.org/10.12973/eu-jer.9.1.129

Introduction

Over the past few decades, there has been a worldwide increase in the development of teacher preparation programs. According to the National Council for Accreditation of Teacher Education "there are two components that prove critically important in teacher preparation: teacher knowledge of the subject to be taught and knowledge and skill in teaching that subject" (National Council for Accreditation of Teacher Education, 2012, p. 4). Ensuring the presence of these components requires renewed effort to better prepare pre-service teachers and align coursework to support them in meeting these requirements. Although the construction of teacher preparation programs varies (Villegas-Reimers, 2003), the majority comprise courses and experiences that address subject matter, educational courses, pedagogical and method courses, and field experiences and clinical practice (Ben-Peretz, 1995; Cobb, 1999). These courses are supposed to provide pre-service teachers with the knowledge and skills essential for their becoming effective teachers in the future.

According to Cockcroft (1982), problem solving lies at the heart of mathematics education, a description that emphasizes the importance of students' acquiring problem-solving skills. The concept of "problem solving" has been defined as "a process by which the learner combines previously learned elements of knowledge, rules, techniques, skills and concepts to provide a solution to a novel situation" (Orton, 1992, p. 35). Other researchers define the problem as "a situation that confronts a person, that requires resolution, and for which the path to the solution is not immediately known" (Posamentier and Krulik, 1998, p. 1). In recent years, considerable attention has been given to the place of

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problem solving in mathematics and the question of how to help students become problem solvers. In addition, The National Council of Teachers of Mathematics (NCTM, 2000), in their Principles and Standards for School Mathematics, identifies problem solving as one of the fundamental process standards that students should achieve. According to the National Research Council (2001), teachers must first possess the professional skills needed to develop students' problem-solving abilities in order to achieve this goal. Teachers also need to counter challenges, such as word problems related to specific mathematical knowledge, and cultivate their own problem-solving strategies and techniques (National Research Council [NRC], 2001).

The techniques involved in solving problems are more closely examined by Polya (1957), who identifies four stages, namely: understanding the problem, planning for solution, carrying out the plan and testing the results. In the first stage, the student needs to understand the words used in stating the problem, identify what he/she is asked to find or show, restate the problem in his/her own words, think of a picture or diagram that might help in understanding the problem, and then examine whether there is enough information to find a solution. In the "planning for solution" stage, the student needs to choose an appropriate strategy to solve the problem. The problem-solving strategy at this stage has been defined by Van de Walle (2004) as the development of a method to solve a problem at hand. Posamantier and Krulik (1998) argue that learning problem-solving strategies and methods and how they are used helps learners to be good problem solvers. A review of the existing literature reveals many different problem-solving strategies identified by researchers, including guess and check, look for a pattern, solve a simpler problem, make a drawing (visual representation), work backwards, organize data in a table or list, use arithmetic operations, use logical reasoning, use symmetry, use a model, use a formula, and solve an equation (NCTM, 2000; Polya, 1997; Posamantier and Krulik, 1998). These strategies fit the framework of the current study and can be used to describe problem-solving strategies demonstrated by the participants. It is worth noting that once the participant has come up with a plan for solution, carrying out that plan is usually considered to be easier than devising it. In general, all that is needed is care and patience with necessary skills, persistence with the chosen plan, and sufficient knowledge to choose another plan if the first does not work. In the final stage, the participant examines the solution they obtained.

Developing a student's problem-solving strategies requires careful instruction because problem solving requires students to formulate specific mathematical problems and acquire different skills in representing the problem via numerical, symbolic, verbal, or graphical representation (Schoenfeld, 1988). This step reflects students' understanding of the problem and helps them to develop appropriate techniques for reaching a solution. Expert problem solvers can form mental representations, detect mathematical relationships, and develop novel solution techniques (Australian Curriculum and Assessment Reporting Authority [ACARA], 2014; Common Core State Standards Initiative [CCSSI], 2010; NCTM, 2000). Gok and Silay (2010) argued that in order to learn problem-solving strategies and choose the appropriate method for a given situation, students should be aware of what they are doing and why. In this study, the assumption is that problem-solving ability can be improved via problem-solving lessons. The study also aims to examine whether the application of problem-solving lessons will develop student learning of problem-solving strategies.

The need for developing these skills was investigated by Fang (2012), who explored the needs of mathematics teachers regarding mathematics professional literacy through quantitative and qualitative research. The results showed that the competencies of spatial visualization, operating, and problem solving are the most important needs for teachers. The findings also demonstrated a need for developing teacher competencies in abstracting and generalizing, using mathematical language and symbols, and improving knowledge of common core mathematical concepts and ideas. In addition, teachers' needs included the ability to apply multiple strategies to solve different kinds of mathematical problems. The same results are supported by Evans (2012), who found that middle and high school mathematics teachers who participated in an alternative certificate program had significant improvement in their problem-solving abilities. However, the study also revealed that although participants showed improvement in problem-solving abilities, these skills were still generally weak: teachers displayed failure to start on a problem, poor literacy skills, and a lack of time or effort in working on their mathematical problems. In another study, Evans (2014) pointed out that in comparison to experienced teachers, less experienced teachers were not well prepared in the content and pedagogy required to teach mathematics successfully. Contrary to Evans (2014), Knight, Wiseman, and Cooner (2000) found that students in professional development schools for pre-service teachers benefit from the new knowledge and activities implemented in mathematics classrooms, which is to say the content and teaching methods used at these schools positively affects elementary students' problem solving. Yet Evans (2014) also argued that one of the best ways to help students to acquire better content understanding is providing them with the opportunity to learn through problem solving and inquiry learning, both of which include critical thinking.

A literature review revealed a complex range of studies that reflect a need to develop mathematics teachers' problem-solving skills and strategies and indicate that these needs are varied and dependent upon specific domains (Bruun, 2013; Evans, 2012; Fang, 2012; Munroe, 2015; Tekin Sitrava, 2018). Thus, this study aims to explore the development of problem-solving strategies in elementary pre-service mathematics teachers via problem-solving classes taught through a mathematics teaching methods course at Qatar University. The study also describes various mathematical problem-solving strategies demonstrated by elementary mathematics pre-service teachers in three different phases of

the research. Consideration of pre-service teachers' problem-solving skills and strategies is crucial in the teaching and learning of mathematics because of the importance of developing such skills in future teachers. Additionally, by analyzing the problem-solving strategies and reasoning processes used to solve problems, important evidence on the thought processes of pre-service teachers can be obtained. The results of this study will help teachers, educators and researchers improve the future development of pre-service teachers' problem-solving strategies.

Literature Review

A review of the literature showed a wide range of studies related to problem-solving skills and strategies for mathematics pre- and in-service teachers involving specific mathematical content or topics. This study builds on the theoretical foundations of existing studies to present research focused on problem-solving strategies. The research collected was reviewed and presented based on participants in those studies addressing teachers' and students' strategies.

Some studies investigated problem-solving strategies demonstrated by elementary pre-service teachers. For instance, Margaret et al. (2012) investigated the problem-solving skills of eight mathematics and science middle-school teachers. Semi-structured interviews were conducted with the eight pre-service teachers after they completed an open-ended triangle task with four unique solutions. The study found that the primary strategy used by pre-service teachers in attempting to solve the triangle problem task was guess and check. The findings also revealed misapplications of guess and check as a systematic problem-solving strategy. The study recommended that teachers' preparation programs give more attention to preparing prospective teachers to teach effectively, particularly in using the guess and check method while solving problems.

In another study, Almeida and Bruno (2014) examined the strategies used by pre-service primary school teachers for solving simple addition problems involving negative numbers. The study results demonstrated six different strategies based on the difficulty of the problem and, in particular, based on the position of the unknown quantity. The findings also revealed that students used negative numbers in easy problems and applied other strategies to more complex problems. Furthermore, the problem-solving strategies addressed difficulties involving negative numbers, such as mistakes with arithmetic rules and a lack of mathematical accuracy in the mathematical notation used in the operations.

Some studies have explored the problem-solving strategies of pre-service secondary school mathematics. For example, Yew and Zamri (2016) investigated the problem-solving strategies of eight pre-service secondary school mathematics teachers. The study followed a case study research design and interviews were used to collect the data. The findings revealed that these teachers used several different strategies, the most common being drawing a diagram, followed by other strategies such as trial and error, identifying patterns, and using an equation or listing. Other, less common strategies used by these pre-service secondary mathematics teachers ranged from primary unsystematic trial and error to a more advanced strategy using prior knowledge of factors, equations, and differentiation. In addition, some pre-service teachers were unable to solve the problems correctly due to their use of limited and incorrect mathematical terminology, lack of problem comprehension, difficulty making representations of the word problems, lack of basic knowledge, and misconceptions regarding the length and area of a rectangle.

Another study investigated problem-solving strategies demonstrated by in-service teachers. More specifically, Bruun (2013) conducted a study for 70 in-service fifth-grade elementary mathematics teachers from 42 schools in a large state in the U.S. Participants were asked to report what strategies were most often used in their attempts to enhance their students' problem-solving skills. The interviews showed that none of the teachers utilized all problem-solving strategies recommended by the National Council of Teachers of Mathematics. Further investigations also revealed that the teachers' most-taught word problem strategy was having their students draw a picture and identify key information. The study indicated a need for clearer instruction in problem-solving strategies when preparing mathematics teachers.

Some studies reviewed here also examined students' problem-solving strategies. In a study reviewing the performance and problem-solving strategies of sixth-grade students considered to be from high-performing counties in East Asia (361 from China and 345 from Singapore), the results indicated that Chinese students used algebraic strategies more frequently and more successfully than the Singaporean students did. The results also revealed that the Chinese students used a limited variety of strategies, while the Singaporean students used a model-drawing strategy. In addition, the Singaporean students displayed a performance advantage on one multiplication/division problem (Chunlian, Stephen, & Jinfa, 2014).

The effectiveness of the teaching approach on students' problem-solving strategies was examined in several studies. For instance, Ramnarain (2014) examined the effectiveness of the strategies-based problem-solving approach on the problem-solving performance of grade nine mathematics classes in South African schools. Using this approach, defined as an instructor gave students clear instructions on problem-solving strategies and used a pre- and post-test control group research design to measure results. The results showed that the students' problem-solving performance improved significantly when a strategies-based approach to problem solving was implemented.

In another study, Al-Balasi and Barham (2010) explored the effect of using multiple mathematical representations in the acquisition of mathematical concepts in grade eight students' ability to solve verbal problems. The study's results demonstrated that developing students' ability to use different mathematical representations enhanced their ability to solve verbal problems and aided in developing their conceptual understanding.

Most studies to date have examined the effect of a teaching approach on students' problem-solving strategies or explored mathematics teachers' or students' problem-solving strategies by having a group of teachers or students solve mathematical problems.. This study innovates in its approach: by exploring the effect of problem-solving lessons, on pre-service teachers' problem-solving strategies. More specifically, this study examines the development of problem-solving strategies for elementary pre-service mathematics teachers via problem-solving mathematics classes delivered in a mathematics teaching methods course at Qatar University. It describes various mathematical problem-solving strategies demonstrated by these teachers in three different phases of the intervention: before, during, and after receiving the mathematics lessons. In particular, the study aims to answer the following two questions.

1. Do problem-solving classes contribute to the development of pre-service elementary teachers' problem-solving strategies?
2. What mathematical problem-solving strategies are demonstrated by pre-service elementary mathematics teachers in different phases of the research?

Methodology

Research Design

The study investigated the development of problem-solving strategies for elementary pre-service mathematics teachers via problem-solving mathematics classes. A mixed quantitative and qualitative research approach was employed by analyzing the collected data throughout different research phases and describing the various mathematical problem-solving strategies demonstrated by elementary pre-service mathematics teachers in each phase. Yin (2008) argued that a qualitative research design is used for in-depth investigation of the current situation in real-life contexts. In addition, Creswell (2007) suggested that qualitative research design is used to explore the solution strategies analyzed throughout different phases of the research. In the current study, the quantitative portion calculates the frequencies and percentage of the elementary pre-service mathematics teachers' responses to the problems posed in three different phases of the intervention: before, during, and after receiving the mathematics lessons. The qualitative approach was used for in-depth investigation to describe various mathematical problem-solving strategies demonstrated by participants across the three different research phases.

Study Group and Data Collection

Study participants consisted of 42 female elementary pre-service mathematics teachers who attended a mathematics teaching methods course (16 weeks) during the first semester of the 2017-2018 academic year at the college of education at Qatar University. The "Participants Convenience" sampling method was utilized to determine the study group (Merriam, 2013). The study group was drawn from the prospective teachers who enrolled in the Bachelor of Education in Primary Education degree program and who had concentrations in math and science. Participants in this program study the goals, methods, and materials available for teaching topics such as numeration, geometry, basic operations, fractions, decimals, percentages, measurement, and probability in the primary school classroom. Issues related to problem-solving skills and strategies are also considered in the courses. All participants volunteered for the study after they had been informed of its objectives.

Data Collection Tools and Procedure

Research followed a case study approach throughout three phases. This approach examined data before, during, and after the implementation of problem-solving mathematics lessons. In each phase of the research, a set of problems was assigned to the study participants and the frequencies and percentages of their responses to the posed problems were calculated. In addition, the collected data was analyzed qualitatively.

In the first phase, at the beginning of the teaching methods course, participants were given a pre-test, which consisted of one problem and aimed to identify the strategies these pre-service teachers used at the starting point. The second phase of the research lasted for two weeks. During this phase, mathematical problems were assigned to the participants to be solved individually and were followed by classroom discussion introducing different problem-solving strategies. In the final phase, a post-test was implemented to explore the development in participants' problem-solving strategies. All problems assigned in this study were generated and modified from the Trends in International Mathematics and Science Study (TIMSS) exams. This is because it is important that pre-service teachers master solving such problems since their future students will be expected to do the same on the international exams. The selected problems were in different subject matter domains. A clear description of the problems is provided in the findings section.

Data Analysis

There are two aspects to the analysis of the teachers' responses, the first being the success of the teachers in solving the problem. The following classification was used to represent their responses.

- Success: the teacher could solve the problem correctly and reach the correct answer.
- Failure: the teacher could not reach the correct answer.
- Ignore: the teacher made no attempt to solve the problem.

Descriptive analysis was applied in calculating the frequencies and percentages of the 42 elementary pre-service mathematics teachers' responses to the problem in each phase of the research. The second aspect of the analysis concentrated on which problem-solving strategies the participants used. These strategies were classified according to the list given in the introduction of this paper (NCTM, 2000; Polya, 1997; Posamantier and Krulik, 1998).

Findings and Discussion

Findings Related to the First Phase of Research

In phase 1 of the research, a pre-test was given to the 42 pre-service mathematics teachers who enrolled in the mathematics teaching methods course. The test consisted of one problem because its aim was merely to explore the initial problem-solving strategies used by the participants. The problem is as follows:

- *A pocket can contain 1/5 kg of flour; how many pockets do we need to fill a bag with 6 kg of flour?*

The participants' responses were analyzed based on their success in solving the problem. The following table represents the frequencies and percentages of these responses.

Table 1. Frequencies and percentages of elementary pre-service mathematics teachers' responses to the problem posed in phase 1.

RESPONSES	Success		Failure		Ignore	
	Freq.	Per. %	Freq.	Per. %	Freq.	Per. %
TOTAL:42	20	47.62	19	45.24	3	7.14

The results in phase 1 revealed that almost half of the pre-service teachers (47.62%) could solve the problem and reached the correct answer, while the others (45.24%) either could not solve the problem correctly or made no attempt to do so (7.14%). This finding demonstrated the weakness of elementary pre-service mathematics teachers in problem solving and emphasizes the need for the current research. This result aligns with others (Bruun, 2013; Margaret et al., 2012; Yew & Zamri, 2016) that address teachers' weakness in problem solving and recommend that teacher preparation programs do more to enhance problem-solving skills.

Analyses of the strategies used by elementary pre-service mathematics teachers in phase 1 of the research also showed a limited application of different problem-solving strategies. Most pre-service teachers who succeeded in solving the problem used the "use arithmetic operation strategy," employing either division or the cross multiplication method, as shown in figures 1 and 2.

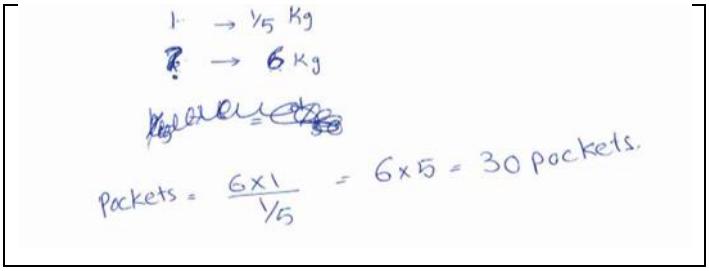

Fig. 1: The "use arithmetic operation" strategy (cross multiplication)

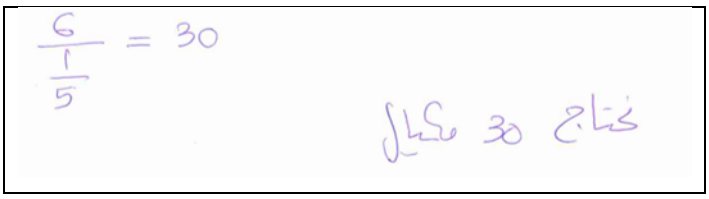

Fig. 2: The "use arithmetic operation" strategy (division).

Analysis of the collected data also demonstrated that although some of pre-service teachers attempted to use other strategies such as "make a drawing," they failed to reach the correct answer. In addition, the results yielded some unreasonable solutions, as shown in figures 3 and 4.

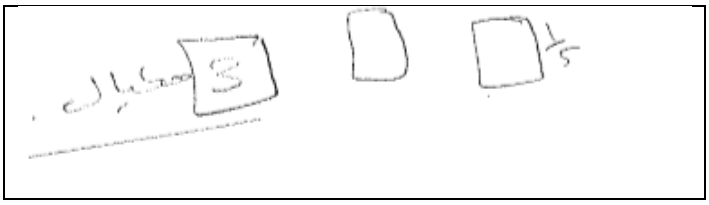

Fig. 3: The "make a drawing" strategy (wrong answer).

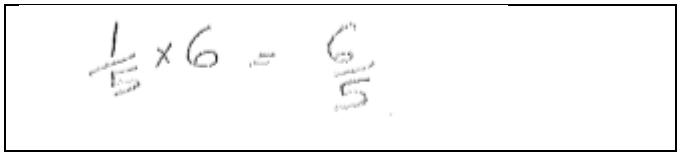

Fig. 4: The "use arithmetic operation" strategy (wrong answer).

Findings Related to the Second Phase of Research

In phase 2 of the research, the mathematical problem-solving classes continued for two weeks. At the beginning of each lesson, a problem was presented to the pre-service teachers who attended the class. They were asked to solve this problem individually. This was followed by classroom discussions designed to introduce different problem-solving strategies and encourage participants to share their solutions. During the classroom discussion, the researcher named each new strategy to provide an opportunity for the pre-service teachers to come to know the different problem-solving methods. The classroom discussion also aimed to recognize that there are different methods and techniques that can be used to solve problems other than just finding a suitable arithmetic operation, thereby guiding participants to the realization that they can use more efficient procedures that may save time and help them reach correct answers. During the classes, different problem-solving strategies were presented based on the type of the problem, such as "use logical reasoning," "solve a simpler problem," "make a drawing," "guess and check," "organize data in a table or a list," "look for a pattern," "work backwards," "solve an equation," and "use arithmetic operations." To demonstrate these strategies, six different problems were introduced throughout the second phase of the research. Here, the findings related to the following three problems will be presented and discussed.

- *Problem 1: Three brothers, Bader, Zaid and Ahmed, received a gift from their father of QR 45,000. The amount was distributed among the brothers by the number of children each. Bader has 2 children, Zaid has 3 children and Ahmed has 4 children. How many QR is each share?*
- *Problem 2: Squares are made by using sticks of the same length, as shown in the figure below. How many sticks are needed to make eight squares in a row? How many sticks are needed to make 100 squares in a row?*

- *Problem 3: What is the value of 10% of 90% of 490?*

  a) 4.41

  b) 44.1

  c) 441

  d) 445.9

Justify your answer.

Table 2 represents the data collected from the responses.

Table 2. Frequencies and percentages of elementary pre-service mathematics teachers' responses to the problems posed in phase 2.

RESPONSES	Success		Failure		Ignore	
	Freq.	Per. %	Freq.	Per. %	Freq.	Per. %
Problem 1 - Total: 39	28	71.79	19	28.21	0	0
Problem 2 - Total: 41	22	53.66	19	46.34	0	0
Problem 3 - Total: 42	29	69.05	13	30.95	0	0

The results in table 2 demonstrate that, in general, the pre-service teachers displayed more success in solving the posed problems during phase 2. The most success was in response to problem 1. This may be due to the type of the problem, as it could be solved based on the concepts of proportion and quantities.

Although there was increased success in solving problem 1, analysis of the strategies used to solve it also showed a definite limitation in this instance, with most participants using the same strategy: adding the quantities that represent the total number of the children, applying the division operation, and then finding the amount for each brother, as shown in figure 5. As seen in figure 6, one participant used the "solve an equation" strategy.

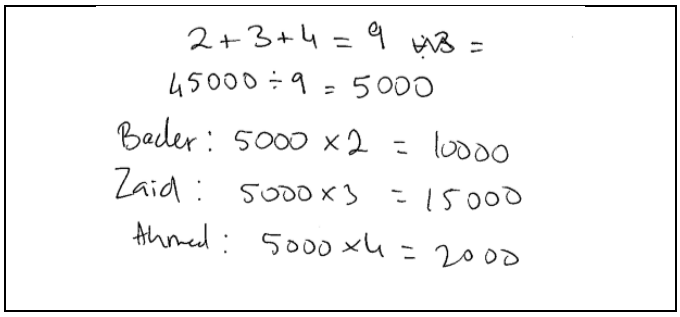

Fig. 5: The "use arithmetic operations" strategy (addition, division, and multiplication).

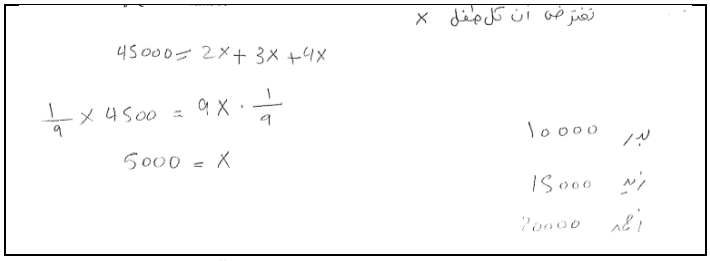

Fig. 6: Use of "solve an equation" algebraic strategy.

Although participants' responses to problem 2 showed less success in solving the problem, rich discussion was observed during the problem-solving class. The discussion allowed the opportunity to introduce different problem-solving strategies, such as "use logical reasoning," "solve a simpler problem," "organize data in a table," "look for a pattern," "solve an equation," and "use arithmetic operations." It was also observed that the pre-service teachers were more enthusiastic about such a problem. Figures 7 and 8 display some examples of strategies used to solve problem 2.

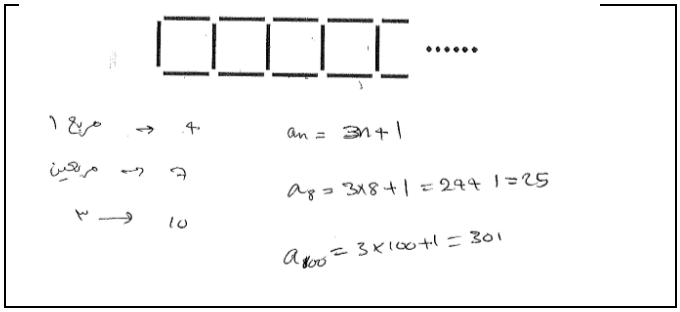

Fig. 7: The "look for a pattern" strategy.

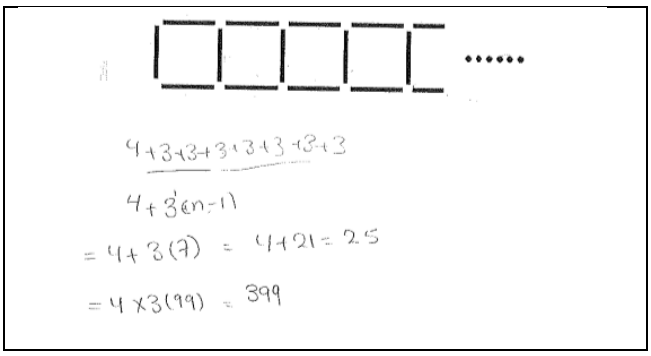

Fig. 8: The "use logical reasoning" and "solve a simpler problem" strategies (wrong multiplication).

The participants' responses to problem 3 showed more success in solving the problem by using arithmetic operations. Other attempts based on estimation and using logical thinking strategies were discussed during the class. Here also, there was rich discussion guiding the participants in thinking logically and making reasonable estimations. An example of such strategies is presented in figure 9.

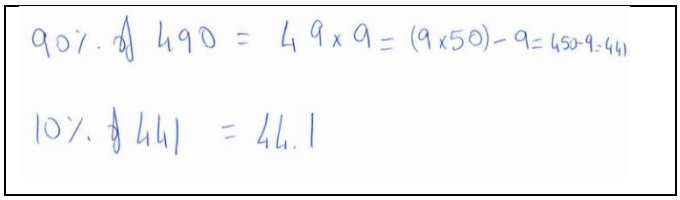

Fig. 9: The "use arithmetic operation" and "use logical reasoning" strategies.

The results in phase 2 of this study aligned with those found by Almeida and Bruno (2014), which indicated that strategies used in solving problems may differ based on the type of the problem. In addition, the results of this study were consistent with that of Ramnarain (2014), who found significant improvement in problem-solving performance when a strategies-based approach to problem solving was implemented. The findings of this study also agreed with those of Al-Balasi and Barham (2010), who concluded that using multiple mathematical representations enhanced students' ability to solve word-based mathematical problems.

Findings Related to the Final Phase of Research

In the final phase of the study, post-tests on problem solving were implemented to explore the development of problem-solving strategies for elementary pre-service mathematics teachers. Based on the definition of problem solving as a new situation that a student must confront, the post-test included the two following problems.

- *Problem 1: A rectangular carpet has an area of 12 m^2 and a perimeter of 14 m. Find the length and the width of the carpet.*
- *Problem 2: Ahmed and Salem started to save money. If Ahmed has 100 QR and saves 30 QR weekly, and Salem has 70 QR and saves 40 QR weekly, after how many weeks they will have the same amount of money?*

Table 3 represents the data collected from the participants' responses.

Table 3. Frequencies and percentages of elementary pre-service mathematics teachers' responses to the problems posed in the final phase.

RESPONSES	Success		Failure		Ignore	
	Freq.	Per. %	Freq.	Per. %	Freq.	Per. %
TOTAL: 42	32	76.19	10	23.81	0	0
TOTAL: 42	34	80.95	8	19.05	0	0

The results shown in table 3 revealed that the pre-service mathematics teachers experienced more success in solving the problems in the final phase of the research than in the first or second phases. While 76.19% could solve problem 1 of the final test, still more participants, 80.95%, had success with problem 2. Furthermore, none of the participants ignored the problems. This finding demonstrates the improvement of elementary pre-service mathematics teachers' problem-solving abilities as a result of learning via problem solving. This result is in line with Ramnarain (2014), who found significant improvement in problem-solving performance when a strategies-based approach to problem solving was implemented.

Participants in phase 3 of the research were also increasingly successful in applying different problem-solving strategies. Most of the pre-service teachers who succeeded in solving problem 1 could use more than one strategy on the same problem. The type of learning approach implemented in the problem-solving lessons the pre-service teachers engaged in were intense and varied. The types of problems given encouraged pre-service teachers to demonstrate different problem-solving methods and techniques. On the other hand, more strategies to solve problem 1 were used in phase 3 than in the first and second phases of the research, reflecting an improvement in participants' command of different problem-solving strategies. These strategies included "use logical reasoning," "make a drawing," "guess and check," "organize data in a table or a list," "solve an equation," and "use arithmetic operations." Some responses are shown in figures 10 and 11.

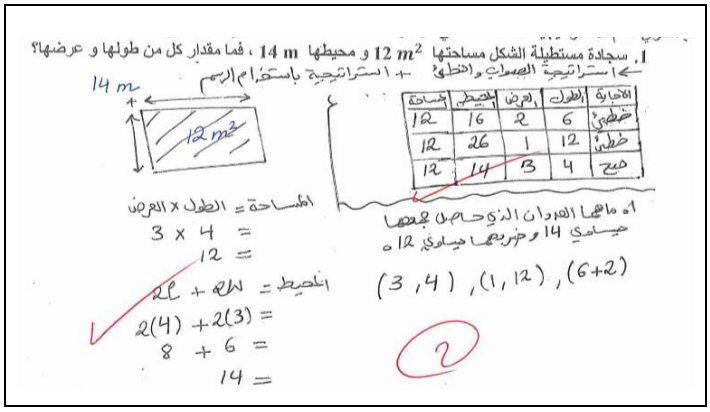

Fig. 10: The "guess and check" and "organize data in a table" strategies.

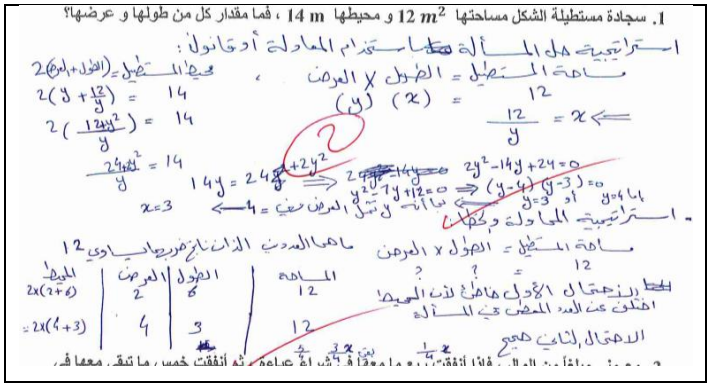

Fig. 11: The "solve an equation" and "guess and check" strategies.

Findings of the study also revealed that some participants were not able to reach the correct answer in problem 1 because they had misconceptions about the differences between the area and the perimeter of the rectangle. Other failed attempts were due to participants' lack of familiarity with the rules of area and perimeter, as shown in figure 12. This result is consistent with the findings of Yew and Zamri (2016), who found pre-service secondary school mathematics teachers had misconceptions regarding the length of a rectangle and its area.

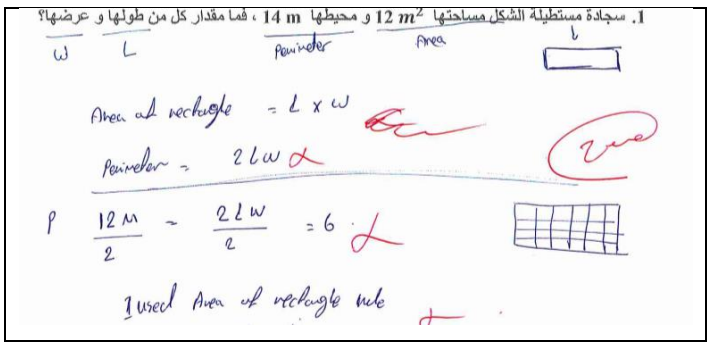

Fig. 12: Lack of familiarity with the rule of perimeter and incorrect arithmetical operations (wrong answer).

Analyzing the problem-solving strategies used in phase 3 of the research also revealed increased success of the pre-service mathematics teachers in applying different problem-solving strategies to solve problem 2. Most of the

participants could use more than one strategy to solve the same problem, suggesting that by being given the opportunity to work on different types of problems during the second phase of the research, by participating in the classroom discussion, and by being taught problem-solving strategies through problem solving lessons, participants' developed and approach they could use to solving problems. More strategies were used to solve problem 2 in phase 3 than were used on problems in the first and second phases of the research, indicating an improvement in participants' problem-solving capabilities. The results in phase 3 of this study were consistent with those of Ramnarain (2014), who found significant improvement in problem-solving performance when a strategies-based approach to problem solving was implemented. In the current study, the results showed an increased number of problem solving strategies being implemented to solve problem 2 in phase 3, such as "use logical reasoning," "guess and check," "organize data in a table or a list," "look for a pattern," "solve an equation," and "use arithmetic operations." Figures 13, 14, and 15 represent some examples of participants' responses.

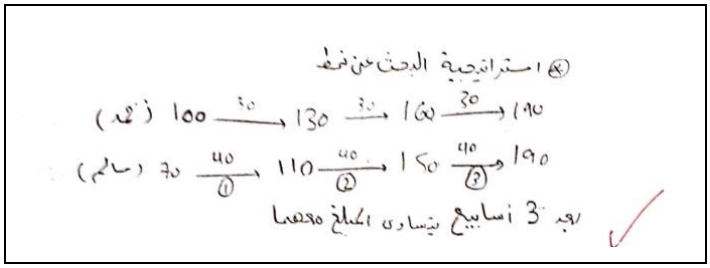

Fig. 13: The "look for a pattern" strategy.

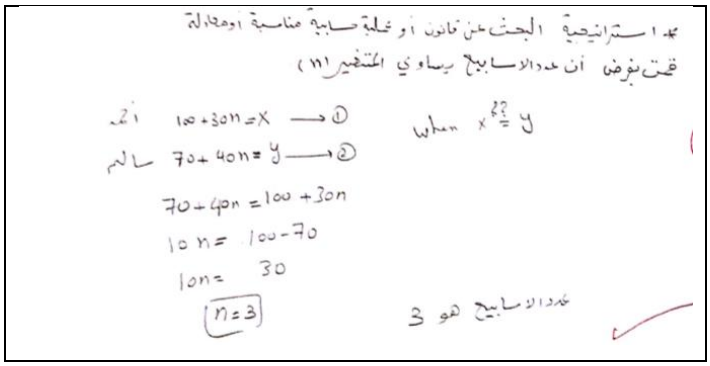

Fig. 14: The "solve an equation" strategy.

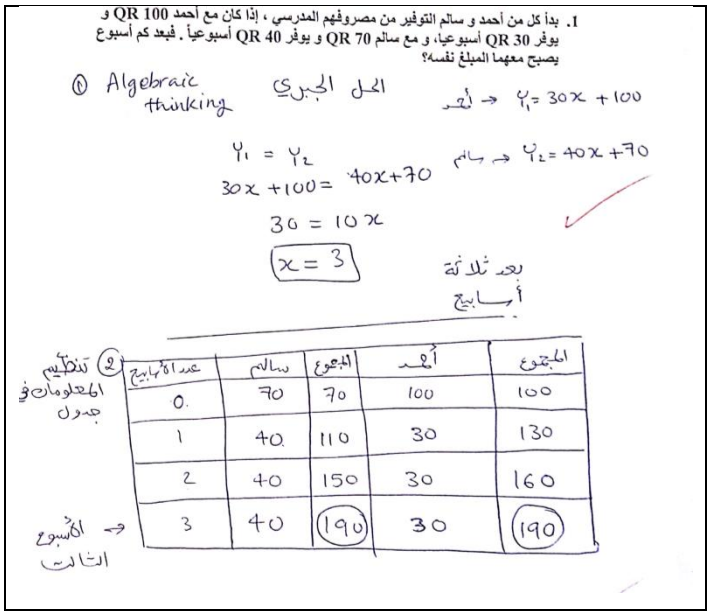

Fig. 15: The "solve an equation" and "organize data in a table" strategies.

The findings also revealed that some of the pre-service teachers were not able to reach the correct answer in problem 2 because they were using limited or incorrect mathematical terminology, lacked understanding of the problem, or had difficulties making representations of the word problem, issues which were also observed by Yew and Zamri in their study (2016). An example of failed attempts is given in figure 16.

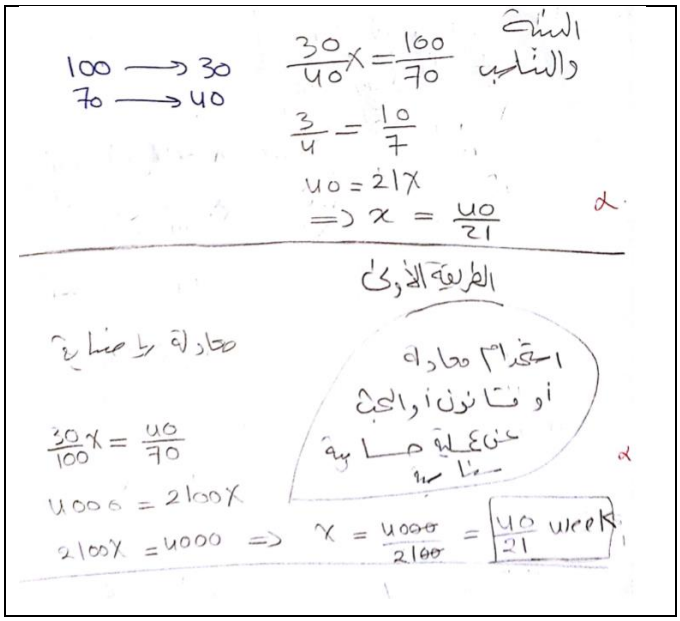

Fig. 16: Lack of understanding and difficulty in making representations of the word problem (wrong answer).

Conclusion and Suggestions

This study aims to investigate the development of problem-solving strategies for elementary pre-service mathematics teachers through problem-solving lessons. The investigation was conducted in three phases over two weeks and describes the strategies utilized in response to problems posed during the three research phases. Findings from the first phase showed a limited number of problem-solving strategies being used by the participants, which emphasizes the need for this study. Following the second phase of the research, which extended over two weeks of problem-solving lessons and classroom discussion, the findings revealed the participants had developed stronger problem-solving skills and were displaying more success in solving mathematical problems. In the post-test in phase 3, the findings indicated that the pre-service teachers had increased their ability to use multiple mathematical representations and that this could help them to develop more problem-solving strategies. The results showed an increased number of varied problem-solving strategies being implemented, such as "use logical reasoning," "solve a simpler problem," "make a drawing," "guess and check," "organize data in a table or a list," "look for a pattern," "work backwards," "solve an equation," and "use arithmetic operations."

This research illustrates the benefits of a problem-solving lessons approach to teacher preparation that supports the development of prospective teachers in mathematical problem solving. However, although this study demonstrated that elementary pre-service mathematics teachers made progress in developing problem-solving skills, it also revealed their weakness in applying the essential skills required for success in solving mathematical problems. This weakness became apparent in their use of limited and incorrect mathematical terminology and displayed lack of understanding of the problem, which both indicate a need for further development in interpreting information, mathematical working, and logical thinking. Based on these findings, the study shows the need for more in-depth research in the area of mathematical problem solving to explore the source of teachers' difficulties and how they might be rectified. Although this study used mixed quantitative and qualitative research approach to investigate various mathematical problem-solving strategies demonstrated by elementary pre-service mathematics teachers in each phase, this research has been limited to the duration of the second phase of the research that extended for 2 weeks. Thus, more time is recommended to develop an in-depth understanding of how problem-solving classes contribute to the development of pre-service elementary teachers' problem-solving strategies.

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