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The PGBE Model for Building Students' Mathematical Knowledge about Percentages

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Abstract: This research study presents the PGBE model for teaching and learning percentages with students of Grade 7 when their cognitive development enables the conceptual understanding of percentages as proportional statements, and offers the possibility for more effective matching of them with fractions and decimal numbers. The abbreviation PGBE presents the interrelation of the poster method and three instructional models through which different types of students' mathematical knowledge about percentages can be built. Hence, P stands for the poster method through which the recognition of students' previous knowledge about percentages can be done, G represents different grids that can be used for building concrete type of knowledge about them; B signifies the bar model for developing students' proportional understanding of percentages, and E represents the extended bar model for fostering students' principled-conceptual understanding of percentages. The effectiveness of the implementation of the PGBE model is assessed by organizing two cycles of piloting and conducting the experimental method with 263 students of ten Grade 7 classes. The results of the study show that the implementation of the PGBE model has had an impact on the learning of students, stimulating an in-depth learning and a long lasting knowledge about percentages for this cohort of students.

Keywords: *Percentage, the PGBE model, design research method, types of students' mathematical knowledge.*

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Introduction

Mathematics is considered a difficult school subject by many students (Gafoor & Kurukkan, 2015; Quintero & Rosario, 2016, Raj Acharya, 2017), but there are some mathematical concepts that easily trigger students' interest and curiosity and percentage is one of them. Percentage has this advantage due to its wide usage in school and in real-life situations. Even though, there is a familiarity of students with percentages before their formal introduction in school (van Galen et al., 2008; van den Heuvel-Panhuizen, 2003), and there is a high interest in learning them during the schooling, many researches have emphasized that students often struggle to solve percentage problems (Parker & Leinhardt, 1995; Scaptura et al., 2007; van den Heuvel-Panhuizen, 2003; van Galen & van Eerde, 2013). The criticism is mainly attributed to the methodology used in teaching percentages, which is primarily focused on recalling procedures for solving percentage problems instead of getting a real understanding of the concept of percentage and its multiple perspectives as a sustainable investment in solving problems related to it (Ngu et al., 2014; Rianasari et al., 2012; van den Heuvel-Panhuisen, 1994; van Galen & van Eerde, 2013). Indeed, methodology that is focused in developing students' skills for solving percentage problems "as a set of data without connection" to their use in different contexts of real-life situations "leads to mechanization and memorization of details" and usually "produce boredom among students, leaving aside their fundamental structure" (Larracilla-Salazar et al., 2019, p. 1039).

There are few publications that promote learning models for supporting students' understanding of percentages (Fosnot & Dolk, 2002; Rianasari et al., 2012; van den Heuvel-Panhuizen, 2003; van Galen et al., 2008). These learning models are designed to introduce percentages in a span of 3 - 4 years starting from Grade 5, when students are still in the concrete operational phase of their development. As percentages are based on proportional reasoning and the ability to reason proportionally is a primary indicator of formal operational thought (Piaget & Inhelder, 1973), there are countries that envision the formal introduction of percentages in upper grades. This research study was focused in designing a new learning model for teaching and learning percentages in Grade 7, when most students enter to the

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formal operation thought and their cognitive development enables the conceptual understanding of percentages as proportional statements, and offers the possibility for more effective matching of them with students' prior knowledge about fractions and decimal numbers as their predecessors. The new learning model support students in gaining a deeper understanding about percentages by building a stronger linkage of them with similar mathematical concepts, such as: fractions and decimal numbers. A stronger link of percentages with fractions and decimal numbers enhance students' skills for using these mathematical concepts interchangeably and implementing the one they are more confident in for solving various problems related to them. As percentages are widely used not only during schooling, but also in real-life situations than there is a need not only to design learning models for supporting students in learning percentages, but also to assess the designed model in a learning context in order to prove its effectiveness. For assessing the effectiveness of the implementation of the new learning model, the empirical study was conducted in Kosovo context.

As there are some quite broad terms related to teaching and learning percentages which are used by researchers in different ways, the definition of some key terms that are used in this study are given below.

Instructional models are "representations of problem situations, which necessarily reflect essential aspects of mathematical concepts and structures that are relevant for the problem situation, but that can have different manifestations" (van den Heuvel-Panhuizen, 2003, p. 13). Instructional models for teaching and learning percentages are: 10 x 10 squares grid, Cuisenaire rods, comparison scales, ratio tables, bar model, extended bar model and double number line.

Learning model is a projected route encompassing learning objectives, strategies, and instructional models that provide meaningful learning opportunities for students to build their understanding of different mathematical concepts in a various learning context. Due to individual differences of students, learning model is a roadmap that guides teachers how to implement instructional activities and models that equip students with different learning capacities to go back and forth during their developmental progression acquired through learning process. Learning model is a rather general concept that resembles the landscape of learning where students, due to individual capacities, can follow different learning paths to achieve learning outcomes (Fusnot & Dolk, 2002).

Methods for solving percentage problems are recognized as a set of procedures that are characteristic for calculating percentages. Methods for calculating percentages are: the three cases method, the formula method, the equation method, the unitary method, the proportional method, ratio tables, etc. (Parker & Leinhardt, 1995; van Galen et al., 2008).

Literature Review

Instructions that teachers use in their classes effect directly students' learning (Confrey & Maloney, 2014; Hodnik-Cadez & Kolar, 2017; Wilson et al., 2015). There are researchers who emphasized that "teachers' content knowledge and changes in teachers' practices both had statistically significant effects on student achievement" (Polly et al., 2015, p. 26), and that "learning achievement of students is a reflection of the teaching quality" (Damrongpanit, 2019, p. 713). For increasing students' achievements, through adjusting teaching to different capacities and styles of students' learning, researchers are recommending teachers to develop a profound understanding of fundamental mathematics (mathematical content) and update their professional knowledge (methodology) for teaching mathematics (Ball et al., 2008), especially in a particular subject area (Clements & Sarama, 2009). In this regard, mathematics educators are designing learning models for teaching and learning percentages as didactic tools for enhancing teachers' professional competences in developing students' learning capacities, mastering their computational skills and sophisticating their mathematical reasoning about percentages.

Learning models, as projected routes for supporting students' learning of percentages, are designed to assist teachers in recalling students' intuitive (or informal) knowledge about percentages and build on it different types of students' mathematical knowledge about them, such as: concrete type of knowledge, procedural (or computational) and principled-conceptual types of knowledge about percentages (Lampert, 1986; van den Heuvel-Panhuizen, 2003). Since the "effective teaching calls for meeting the students where they are and helping them build on what they know" (Clements & Sarama, 2009, p. ix), the formal introduction of percentages usually starts by identifying students' previous knowledge about percentages, follow up with activities that build their concrete level of understanding, continue with the development of their procedural skills for calculating percentages and culminate with grasping principles that percentages fulfill in order to foster the principled-conceptual comprehension of them.

A careful investigation of research studies shows that there are two approaches for designing learning models for teaching and learning percentages. The first approach is in favor of implementing a single instructional model as a tool for developing different levels of students' cognitive thinking about percentages, such as: the usage of 10 x 10 squares grid (or hundreds board) (Bennett & Nelson, 1994; Ningsih et al., 2017; Scaptura et al., 2007), Cuisenaire rods (Erickson, 1990), comparison scales (Dewar, 1984; Haubner, 1992) and bar model (Pohler & Prediger, 2015; Rianasari et al., 2012; van den Heuvel-Panhuizen, 2003). However, there are other researchers who designed learning models for teaching and learning percentages by combining two or more instructional models and methods, those mentioned above as well as additional ones, such as: the double number line, the ratio table, the poster method, etc. (Fosnot & Dolk, 2002; van Galen et al., 2008).

The variety of instructional models for teaching and learning percentages motivated many researchers to analyze the advantages of the implementation of different instructional models in building a particular type of students' mathematical knowledge about percentages (Bennet & Nelson, 1994; Parker & Leinhardt, 1995; Scaptura et al., 2007; van Galen et al., 2008). However, in their review of research studies related to percentages published in a span of seven decades, Parker and Leinhardt (1995) also pointed out the disadvantages of their implementation in addressing a certain issue within the teaching and learning of percentages. Indeed, there are arguments showing that the implementation of the 10 x 10 squares grid is an effective instructional model for building students' understanding of percentage as a part of the whole, in which the whole has 100 equal parts (Bennett & Nelson, 1994; Ningsih et al., 2017), but its implementation has a limited success in developing students' skills in solving problems with percentages greater than 100 %. Parker and Leinhardt (1995) emphasized that "it is not uncommon to see 150 % illustrated by shading all of a hundreds board and half of a second. This representation is a good illustration of 1 1/2 hundreds boards (an extensive quantity) but in no way makes it apparent that the representation is meant to illustrate an area that is 150 % of one hundreds board." (p. 469).

Regarding comparison scale, designed by Dewar (1984), it is a practical tool for mastering students' skills to solve examples that involve fractional uses of percent, but it is not so effective in building students' understanding of percentages in a ratio perspective. Parker and Leinhardt (1995) clearly emphasized that "we doubt that this model would be helpful for those contexts that are ratio comparisons" (p. 467).

As for the bar model, designed by van den Heuvel-Panhuizen (2003), it is an effective instructional model for building the meaning of percentage as a proportional relationship between a part and the whole and for mastering students' skills for solving percentage problems (Pohler & Prediger, 2015; Rianasari et al., 2012; van Galen & van Eerde, 2013), but its implementation at the beginning of introducing percentages and explaining the etymology of the word "percentage" as a part of 100 equal squares is less effective than the implementation of the 10 x 10 squares grid, due to the 100 equal squares this grid contains.

The advantages and limitations of various instructional models in building a certain type of students' mathematical knowledge are important factors that favor the use of the second approach in designing learning models for teaching and learning percentages. The second approach has an advantage, because it offers the possibility to design a learning model for teaching and learning percentages by interrelating different instructional models that are acknowledged for maximizing the development of certain type of students' mathematical knowledge within the overarching knowledge related to percentages.

Methodology

Research goal

The goal of this research study was to design a new learning model for building students' understanding of percentages and assess its effectiveness in the Kosovo context. The new learning model would encompass learning objectives, strategies, and instructional models in a novel, unique way that provide meaningful learning opportunities for students to build their understanding about percentages in a learning context. As the new learning model intended to construct students' knowledge about percentages from their previous experiences and through social interaction between teacher and students and also between students themselves, the social-constructivism approach was used as the underpinning philosophy to lay out the ground and direct the design of it.

Methodology

The design research method, based on teaching experiment, was used for designing the new learning model for teaching and learning percentages. This method offered the possibility to use the inductive approach in designing initially two Hypothetical Learning Trajectories (HLTs), and pilot them consecutively as teaching experiments in two different classrooms and use their feedback to refine them into a new learning model for teaching and learning percentages. For assessing the effectiveness of its implementation, the experimental method was used. Research questions that guided the research work were:

1. How should instructional models be integrated into a learning model in order to support students' learning of percentages?

2. To what extent does the designed learning model support understanding of percentages by 13 and 14-year-old students (Grade 7)?

Sample

The research sample was constituted from 263 students of ten Grade 7 classes and ten mathematics teachers who taught them from three large municipalities of Kosovo. From the research sample, there were 49 students who belonged to two classes where the consecutive piloting of the HLT 1 and HLT 2 took place respectively, and 214 students of eight Grade 7 classes from two other municipalities, who participated in the experimental method of the research study.

Piloting

There were 49 students who belonged to two classes (classes A and B) of two different schools of one municipality, where the consecutive piloting of the HLT 1 and HLT 2 took place respectively. The feedback from piloting was used to finalize the new model for building students' understanding of percentages.

The experimental method

There were eight schools, four schools per municipality that were selected randomly from two other municipalities to participate in the experimental method. To ensure the equivalence of students' mathematical knowledge for participating in the experimental method, from each of eight selected schools it was chosen one Grade 7 class with similar results in mathematics. The average of students' grades in mathematics in Grade 6 of eight selected classes was between 3.55 and 3.60 out of 5. Four Grade 7 classes (two classes per one municipality) constituted the experimental group and the other four classes of Grade 7 (two classes per one municipality) belonged to the control group. Number of students per group and number of students based on gender is given in Table 1.

Table 1. Number of students per group and number of students based on gender.

	Number of students	Number of females	Number of males
Experimental group	104	54	50
Control group	110	55	55
Total	214	109	105

All teachers who taught in both groups had the same qualification, the Bachelor's degree in mathematics and the national regular career license for teaching mathematics in the lower secondary schools. Some characteristics of teachers that taught in the experimental and control groups respectively are given in the table below.

Table 2. Some teachers' characteristics that taught in the experimental and control groups.

	Mean age of teachers	Average years of professional experience
Experimental group	47.00	24.25
Control group	46.75	24.50

Four teachers of classes of the experimental group attended a 3-day training for implementing the new learning model in their classes. After the training, four teachers of classes of the experimental group implemented the new model in their classes for two and half weeks, whereas four teachers of classes of the control group organized the teaching and learning of percentages based on curriculum framework, textbook instructions, and their experience for the same period of time. After the introduction of percentages through the implementation of the new learning model in classes of the experimental group and through curriculum framework and textbooks instructions in classes of the control group, the knowledge test on percentages was organized for 214 students of both groups. For assessing the sustainability of the knowledge gained a retention test on percentages was organized for both groups of students two months later.

Research instruments

Two teachers who piloted the HLT 1 and HLT 2 respectively and four teachers who implemented the new model in classes of the experimental group were provided with lesson plans for teaching and learning percentages. For assessing students' knowledge about percentages, two instruments were designed: i) the knowledge test (see Appendix) and ii) the retention test, which was similar to the knowledge test. The knowledge test was piloted twice in order to ensure its reliability, whereas the retention test was designed by following the same typology of problems as the knowledge test. The Cronbach's alpha of the knowledge test in the first piloting with 20 students selected randomly was 0.911, whereas the Cronbach's alpha during the second piloting with 49 students of two intact Grade 7 classes was 0.831. Each of the knowledge test and retention test contained 16 problems with 28 items in them clustered in four categories for assessing four types of students' mathematical knowledge (Table 3).

Table 3. Clustering of problems of the knowledge and retention tests into four categories for building four types of students' knowledge.

Intuitive knowledge	Concrete type of knowledge	Procedural knowledge	Principled-conceptual knowledge
Problems 1-2	Problems 3-5	Problems 6-11	Problems 12-16

Analyzing of Data

For assessing the piloting of the HLT 1 and HLT 2 in classes A and B respectively different instruments were used, such as: classroom observation protocol, students' exit cards, teachers' reflective diaries and students' results in the knowledge test on percentages. Critical reflection on classroom observation protocols, on students' and teachers' opinions regarding the learning process of percentages during the implementation of the HLT 1, and statistical analysis of students' results in the knowledge test on percentages conducted through the SPSS program were used to assess the implementation of the HLT 1. The triangulation of data confirmed the interrelation of the instructional models within the HLT 1, but suggested the rearrangement of the methods for solving percentage problems within it. As a result of incorporating the feedback from the implementation of the HLT 1, the HLT 2 was designed. The same procedure was used to assess the implementation of the HLT 2. The feedback from the implementation of the HLT 2 was used to design the new learning model for building students' understanding of percentages.

After the introduction of percentages, through the implementation of the new model in classes of the experimental group and based on curriculum framework and textbooks in classes of the control group, 214 students of the experimental and control groups completed the knowledge test on percentages. For assessing the sustainability of the acquired knowledge about percentages, the same students participated in the retention test conducted two months later. The statistical analysis of students' results in both tests was done by using the SPSS program. Students' results in the knowledge test and retention test fulfilled the assumptions of the parametric tests, so the independent t-test was used to perform statistical analysis of students' results in both of them, whereas for performing statistical analyses of students' results in the knowledge test on percentages related to four types of mathematical knowledge the Mann-Whitney U test was used.

Results

There are two results within this study: 1) the PGBE model for building students' mathematical knowledge about percentages and 2) the results from the experimental method conducted for assessing the effectiveness of its implementation in the Kosovo context.

1. *The PGBE model for building students' mathematical knowledge about percentages*

Due to the complexity of designing a new learning model for teaching and learning percentages, initially the framework of it was set up. The framework encompasses a cluster for identifying students' previous knowledge about percentages and three other clusters related to learning objectives of percentages (Figure 1), such as: i) constructing the context-based meaning of the percentage as a part of the whole, ii) interpreting and solving percentage problems, and iii) using percentage as an operator for solving percent decrease and increase problems (Parker & Leinhardt, 1995; van den Heuvel-Panhuizen, 2003).

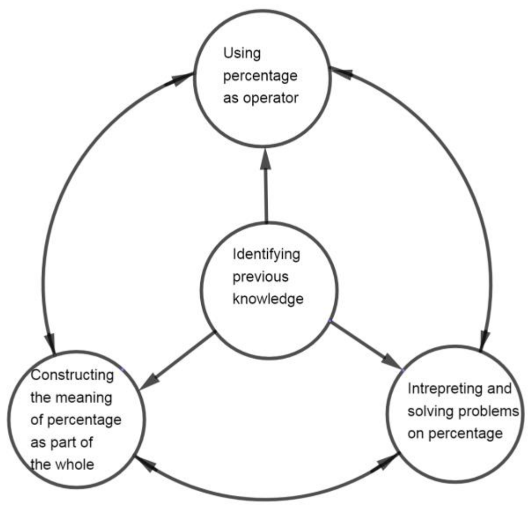


Figure 1. Clusters related to learning objectives of percentages.

Since students are familiar with percentages before their introduction in school (van den Heuvel-Panhuizen, 2003; van Galen et al., 2008), in the center of the framework is a cluster that refers to the identification of students' previous knowledge about them. Students' previous knowledge about percentages can be multifaceted; it can be related to the meaning of the concept as part of the whole, the implementation of different procedures for solving percentage problems as well as the use of percentage as an operator for solving percent decrease and increase problems, so the cluster in the center of the framework is connected and feeds three other clusters of learning objectives of percentages (Figure 1).

Since many recent studies are supporting the idea that students do not all think in the same way, and the paths of student's learning are not necessarily linear (van den Heuvel-Panhuizen, 2003; van Galen et al., 2008), but they usually twist and turn (Fusnot & Dolk, 2002), the achievement of three learning objectives of percentages follow a cyclic process, which allows the possibility for students to go forth and turn back in achieving knowledge related to these clusters (Figure 1).

Determining what is important to learn (learning objectives) and what types of students' knowledge have to be developed through learning imposed the necessity to identify the most effective instructional models for matching both parts. By analyzing the advantages and limitations of all instructional models for building four types of students' mathematical knowledge, it is considered that the method and instructional models that are recognized in the literature as the poster method, 10 x 10 squares grid, bar model and extended bar model (Bennet & Nelson, 1994; Logan et al., 2015; van den Hauvel-Panhuisen, 2003; van Galen et al., 2008) are the most effective ones for recalling the intuitive knowledge and building on it the concrete type, procedural and principled-conceptual type of knowledge about percentages. The interrelation of the most effective research-based instructional models for building in a continuous way four types of students' mathematical knowledge about percentages was done within the development path of the HLT 1 (Table 4). After recalling students' previous knowledge about percentages through the poster method, which consist in giving students a particular task for developing a concept map or listing all of their knowledge about percentages, students have to learn about percentages and implement in practice what they have learned about them. Wells (2016) emphasized: "ideally, pupils need to appreciate that what they are learning can be used for some worthwhile purpose, that they can do something with it - solve a problem" (p. 14). For this reason, the implementation of the instructional models for building different types of students' knowledge about percentages is associated with the use of methods for solving different kinds of percentage problems. The use of methods for solving percentage problems within the instructional models, such as: Unitary method, Formula method, Proportional method, etc. within the HLT 1 is given in Table 4.

Table 4. Learning objectives, types of knowledge, the interrelation of instructional models and methods for teaching and learning percentages through the HLT 1.

Teaching episodes	Learning objectives	Type of knowledge	The sequence of the instructional models of the HTL 1	Methods for solving percentage problems[†]
1.	Identifying previous knowledge	Intuitive knowledge	Poster method	
2. 3. 4.	Constructing the meaning of percentage as a part of the whole	Concrete type of knowledge	10 x 10 squares grid	Unitary method Ratio table Unitary method
5. 6. 7.	Interpreting and solving three types of percentage problems	Procedural knowledge	Bar model	Proportional method Formula method Three cases method
8. 9. 10.	Using percentage as an operator for solving percent decrease and increase problems	Principled-conceptual knowledge	Bar model Extended bar model	Proportional method

The piloting of the HLT 1 was done in a Grade 7 class codded as class A within an allocated time of two and half weeks consisting of 10 class periods. The feedback from its implementation confirmed the interrelation of the instructional models, but imposed the necessity to rearrange the order of the methods for solving percentage problems within the development path of the HLT 2. After piloting the HLT 2 in a different classroom setting codded as class B and getting feedback from its implementation, the answer of the first research question regarding the effective interrelation of instructional models within the new learning model (the PGBE model) and the methods for solving percentage problems is presented in Figure 2.

[†] Terminology for methods for solving percentage problems is used on the basis of the literature.

The PGBE model has two parts: i) the basement consisting from the clusters related to learning objectives of percentages and ii) the development path (presented through arrows) for developing ideas of learning percentages in order to build different types of students' mathematical knowledge about them. The abbreviation PGBE presents the interrelation of the poster method and three instructional models within the development path of the learning model. Hence, P stands for the poster method through which the recognition of students' previous knowledge about percentages can be done, G represents different patterns of grids for building concrete type of knowledge about them; B signifies the bar model for developing students' proportional understanding of percentages and building procedural knowledge for solving problems related to them, and E represents the extended bar model for fostering students' principled-conceptual understanding of percentages.

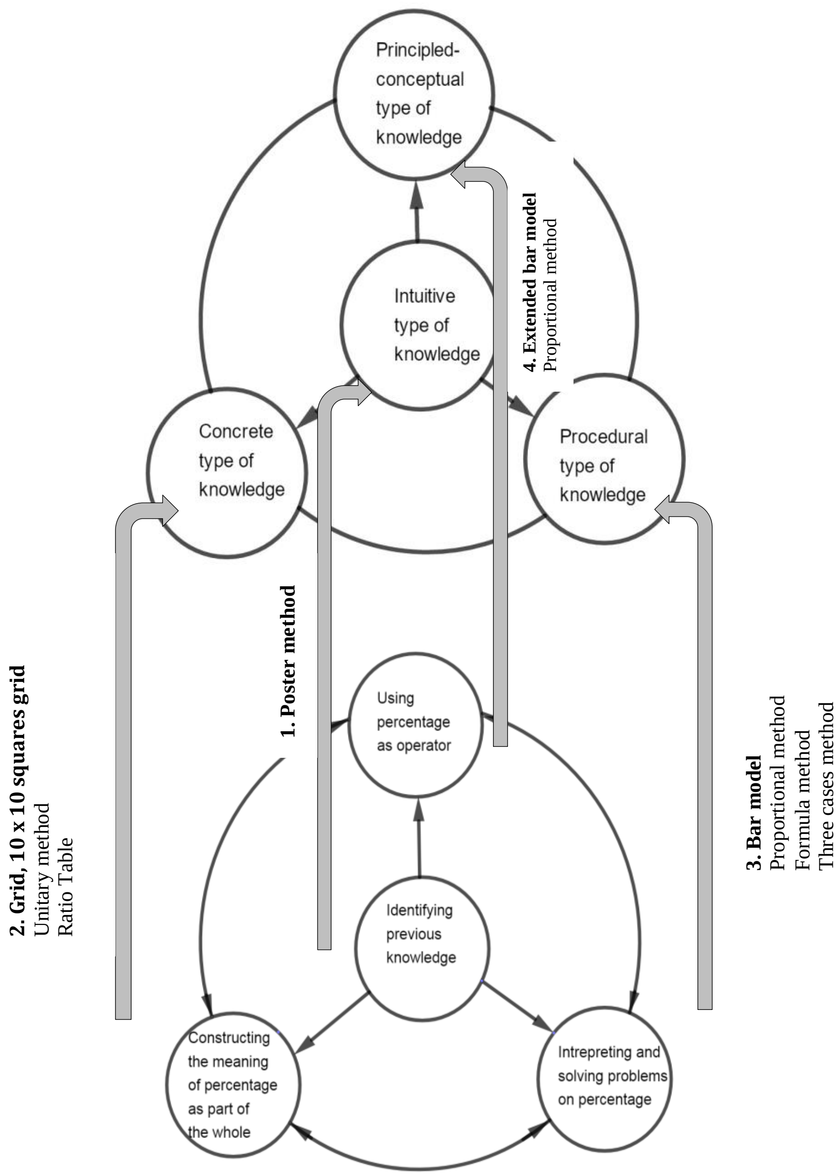


Figure 2. The PGBE model for building students' mathematical knowledge about percentages.

Implementation of the PGBE model starts from the cluster in the center that refers to identifying students' prior knowledge about percentages. The poster method, which requires students to clip different articles from newspapers and journals and create a story based on them, is considered as the most creative and effective method to engage students in recalling their previous knowledge about percentages and improve their metacognitive practice about them (Logan et al., 2015).

The second cluster of the basement incorporates the learning objective for constructing the meaning of percentages as part of the whole. Since the etymology of the name percentage is originated from the expression "per cent" meaning per hundreds, then the usage of 10 x 10 squares grid is a proper instructional model for building students' concrete level of understanding about percentages in a fraction sense as part of the whole and explaining in the best way the meaning of the word percentage as something out of 100 equal parts (Bennett & Nelson, 1994; Ningsih et al., 2017; Parker & Leinhardt, 1995; Scaptura et al., 2007).

Different from Bennett and Nelson's (1994) idea for shading parts of 10 x 10 squares grid for introducing percentages in a visual way as something out of 100 equal parts, the PGBE model contains its improved version suggested by Scaptura et al. (2007) for designing mosaics with 3-6 different colors in the 10 x 10 squares grid. The curiosity to identify percentages of colored parts of their mosaics and fill in the table below (Figure 3) support students to develop their *sense* of percentage as part of a whole, as "something out of 100 equal pieces" and also to link them with decimal fractions and decimal numbers.

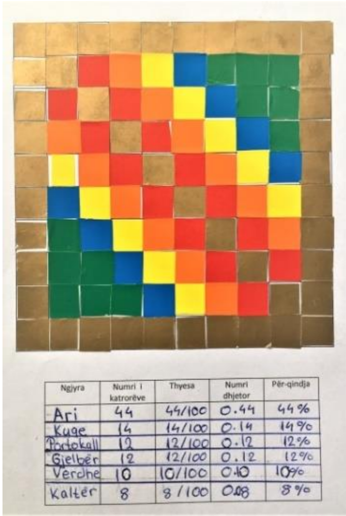


Figure 3. Students' individual mosaic

Additionally, mosaics built in 10 x 10 squares grids are practical tools for transmitting important messages to students, such as: i) their "products" are useful for learning percentages, ii) percentage represent a part of the whole, in which the whole has 100 equal parts, iii) 1 small square is 1/100 of the mosaic and it represents 1 % of the whole, iv) 10 small squares covers 1/10 of the mosaic and they represent 10 % of the whole, v) percentages are linked with fractions and decimal numbers, vi) mathematics can be learned from pieces of visual art, etc. Similar to working in 10 x 10 squares grid, students can work in other grids, such as: 5 x 10 and 5 x 5 squares grids and "reinvent" the usage of equivalent fractions as a tool for converting fractions into percentages and build a mutual linkage of them (Ningsih et al., 2017; Scaptura et al., 2007). Squares grids are also effective tools for guiding students in adding, subtracting and comparing percentages when they refer to the same whole and in building their understanding that additional operations need to be done when the whole differs, such as in case of adding, subtracting and comparing fractions with different denominators.

The initial students' additive thinking and reasoning about percentages, developed through 10 x 10 square grids as so-much-part of the whole in which the whole has 100 equal parts, has to be developed further in understanding the multiplicative perspective of percentages as a proportional relationship between part and the whole, in which the whole might differ from 100 (Dole, 2008; Parker & Leinhardt, 1995; van den Heuvel-Panhuizen, 2003).

Unlike from Bennett and Nelson (1994) who suggested continuing with 10 x 10 squares grid for moving students' understanding from the additive to the multiplicative character of percentages, the PGBE model recommends the usage of ratio tables as more effective instructional model for doing it. Van Galen et al. (2008) emphasized that ratio tables are important tools to develop students' proportional thinking and to master their skills in calculating percentages due to the possibility they offer to run iterative calculations and find percentages. Being familiar with the meaning of 10 % from working with 10 x 10 squares grid, students can easily use the ratio table and calculate some percentages, for example, 30 % of 360. Supported by ratio table, students can easily run iterative calculations and find that 10 % of 360 equals 36, then 20 % of it equals 72, and finally calculate that 30 % of 360 equals 108 (Table 5).

Table 5. Using ratio tables for calculating percentages.

amount	360	36	72	108
percentage	100 %	10 %	20 %	30 %

The implementation of ratio tables in learning percentages assist students to comprehend easily percent benchmarks, such as 1 % -, 5 % -, 10 % - benchmarks and master their usage in calculating percentages. The understanding of 1 % -

benchmark offers students' the possibility to use the unitary method in calculating percentages through the formula $a\,\% = a\ \cdot 1\,\%$. As ratio tables are not always related to a concrete-context situation, they are effective tools to bridge students' concrete and abstract meaning of percentage as a proportional relationship between a part and the whole.

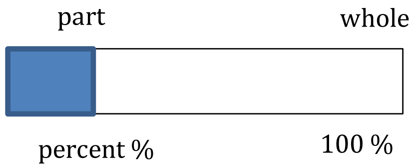


Figure 4. Bar model

Bar model (Figure 4) is the most effective tool for strengthening the meaning of percentage as a proportional relationship and for developing students' critical thinking for interpreting and solving percentage problems. The visual aspect of the bar model is a strong feature that support students to "correctly analyze the referents" (part, percent and the whole) and "understand the proportional relationship that exists between the referent quantities" (Parker & Leinhardt, 1995, p. 463). Since the bar model has a "body area", it assists students to develop their mental schema that "so much part from the whole" corresponds to "this much percent from 100 %" and thus build their proportional reasoning that characterizes percentages (van Galen et al., 2008, p. 93). Due to the double scale, students easily position corresponding quantities of percentage problems above bar model in the top scale and affiliated percentages below it in the bottom scale (van den Heuvel-Panhuizen, 2003; van Galen et al., 2008). The positioning of referents in the bar model assist students to easily write proportional equations $\frac{part}{whole} = \frac{percent}{100}$ and solve them through the implementation of the algebraic procedure and the Rule of Three. Teachers should instruct students to use the algebraic procedure for solving proportional equations and use the Rule of Three only when they intend to automatize the process of solving proportional equations for students and free up their mental space that is required for solving percentage problems (Sweller & Low, 1992). In supporting this idea, Streefland (1985) stated that "the rule of three and cross-product may make a provisional endpoint in this learning process" (p. 92).

By manipulating referents of percentage problems in the proportional equations, students can master the formula method $P_{\text{percentage}} = R_{\text{rate}} \cdot B_{\text{base}}$ and three cases method for solving three types of percentage problems:

- Type I: Finding a part or percentage of a number (e.g., 25 % of 20 is x)
- Type II: Finding the percent when the number and its percentage are known (e.g., x % of 15 is 5)
- Type III: Finding the number when the percentage and its' percent are known (e.g., 20 % of x is 6).

The fourth cluster of the learning process addresses the interpretation and solution of percent decrease and increase problems. The bar model enables students to solve percent decrease problems by using the two-step method (finding the percentage of the whole and subtracting the result from it) or the one-step method by implementing the complement principle, where instead of calculating 15 % decrease, they calculate 85 % of the whole as in Figure 5.

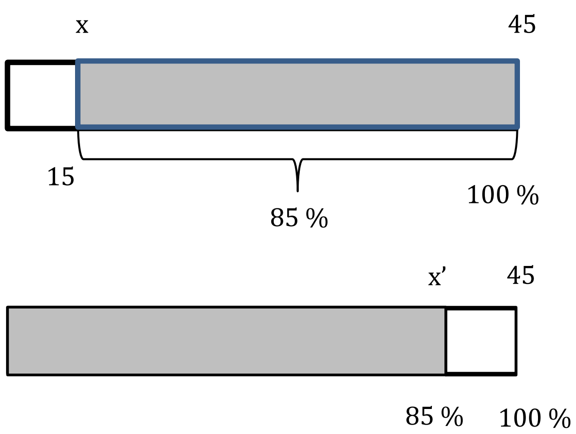


Figure 5. Solving percent decrease problems through the bar model.

For solving percent increase problems, teachers are suggested to use the extended bar model (Figure 6) and its thinner version, the double number line (Figure 7). These instructional models assist students to position referents of percent increase problems in their both scales as in case of the bar model. But, students should pay additional attention in positioning referents of percent increase problems in the right side of 100 % and in interpreting them.

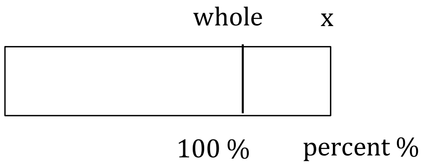


Figure 6. Extended bar model.

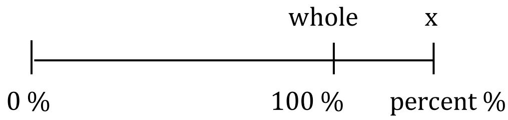


Figure 7. Double number line.

The implementation of the one-step method in solving percent decrease and increase problems enables the usage of percentage as an operator. In these problems, when there is a 15 % decrease or 15 % increase, "the original price (OP)" and "the final price (FP)" are related with a linear equation with the coefficient less than 1 in case of percent decrease problem OP × 0.85 = FP and with the coefficient more than 1 in case of percent increase problem OP × 1.15 = FP (van den Heuvel-Panhuizen, 2003).

Since Poster method, Grid (10 x 10 squares grid), Bar model and Extended bar model are used for introducing percentages, for easy recognition the new model is called the PGBE model for building students' knowledge about percentages. The interrelation of the poster method and three instructional models within the PGBE model for teaching and learning percentages is done for the first time. The PGBE model guides teachers how to implement instructional models and activities for achieving learning objectives of percentages through supporting the developmental progression of students in building their knowledge about them.

2. *The effectiveness of the implementation of the PGBE model*

The effectiveness of the implementation of the PGBE model for building students' mathematical knowledge about percentages was assessed through the experimental method. After implementing the PGBE model in classes of the experimental group and teaching percentages based on curriculum instructions and textbooks in classes of the control group, the knowledge test on percentages was organized on the same day in each of two municipalities for both groups of students. The knowledge test was organized for 214 students of eight Grade 7 classes, from which 104 students of four Grade 7 classes belonged to the experimental group and 110 students of other four Grade 7 classes belonged to the control group.

As students' results in the knowledge test fulfilled assumptions of the independent samples t-test (Table 6), such as: data were normally distributed, there was the homogeneity of variance between the experimental and control groups, and data were measured at the interval level, the mean is used as a statistical model for interpreting students' results of both groups.

Table 6. Independent sample t-test for comparing means of students' results of the experimental and control groups in the knowledge test.

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	**Sig.**	**t**	**df**	**Sig. (2-tailed)**	**Mean Difference**	**Std. Error Difference**	**Lower**	**Upper**
Sum	Equal variances assumed	1.082	.299	7.606	212	.000	2.28051	.29982	1.68950	2.87151
	Equal variances not assumed			7.626	211.741	.000	2.28051	.29905	1.69101	2.87000

Based on the independent samples t-test, students of the experimental group achieved significantly better results in the knowledge test (M = 15.48, SE = 0.20), than students of the control group (M = 13.20, SE = 0.22) (Table 7).

Table 7. The mean and standard deviation of students' result in the knowledge test.

	Class	N	Mean	Std. Deviation	Std. Error Mean
Sum	Exp. group	104	15.4760	2.08715	.20466
	Cont. group	110	13.1955	2.28687	.21804

This difference is significant t (212) = 7.61, p < .05 and it represent a medium size effect r = .46. The knowledge test had 22 points in total, so students of the experimental group were 70.35 % successful in the knowledge test, whereas their counterparts were successful 59.98 % in it. An additional confirmation that the learning of percentages was more effective for students of the experimental group is proved by students' results in the retention test, which took place two months after the introduction of the PGBE model in classes of the experimental group. The independent samples t-test shows that students of the experimental group achieved significantly better results in the retention test (M = 14.66, SE = 0.32) than students of the control group (M = 13.25, SE = 0.32) (Table 8). The difference in the retention test is significant t (211) = 3.15, p < .05 and represents a small size effect r = .21.

Table 8. The mean and standard deviation of students' result in the retention test.

	Class	N	Mean	Std. Deviation	Std. Error Mean
Sum	Exp. group	104	14.6635	3.27349	.32099
	Cont. group	109	13.2431	3.30657	.31671

Although there is a statistically significant difference between students' results of the experimental and control groups in the knowledge test and in the retention test, the low advantage of students of the experimental group in both tests is a result of many factors that influenced them, such as: i) the difficult situation in which the system of education is going through in Kosovo (Shala & Grajcevci, 2018), ii) a short period of time for embedding in teachers the novel approaches for teaching percentages, and iii) a short period of time for adjusting students to the new way of learning.

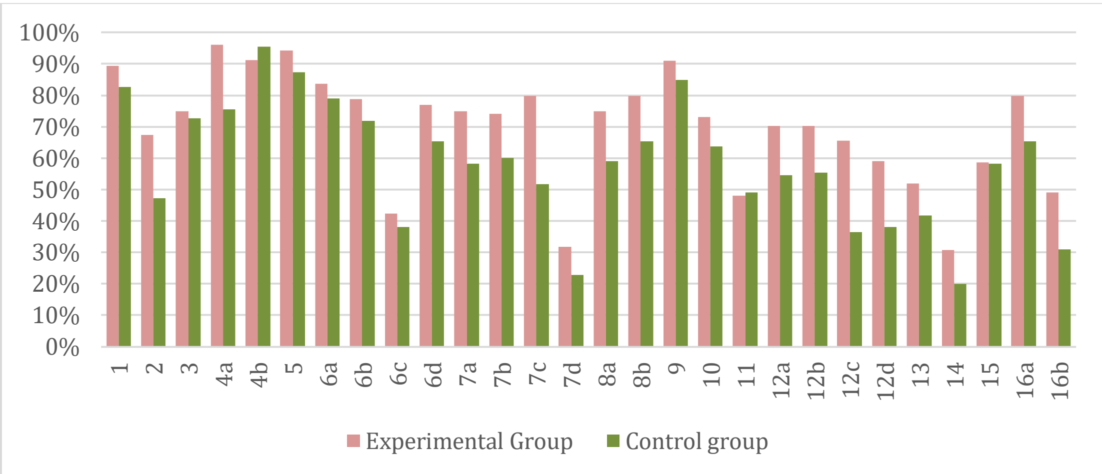


Figure 8. Students' correct answers of both groups in the knowledge test.

A detailed analysis of students' results in the knowledge test on percentages (see Appendix) and in the retention test on percentages designed with similar problems as the knowledge test shows that students of the experimental group outperformed students of the control group in both tests. Students' results of the experimental group are higher in 26 out of 28 items of the knowledge test (Figure 8), and in 21 out of 28 items of the retention test (Figure 9).

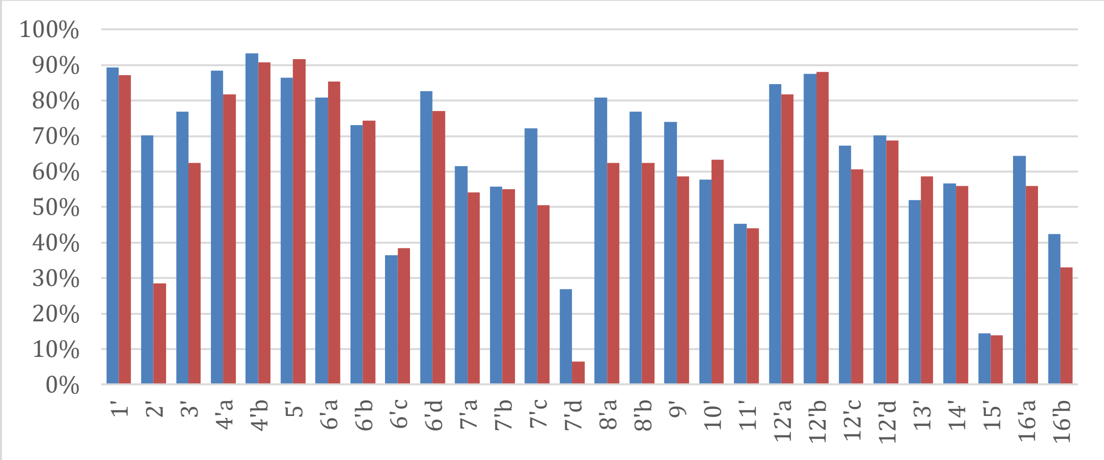


Figure 9. Students' correct answers of both groups in the retention test.

2.1. Analysis of students' results regarding four types of mathematical knowledge

The 16 problems of each of the knowledge test and retention test were clustered in four groups in a similar way. Each group contained problems for assessing one of four types of students' mathematical knowledge about percentages. Problems 1-2 belonged to cluster related to the intuitive knowledge, Problems 3-5 were part of cluster for assessing concrete type of knowledge, Problems 6-11 belonged to cluster related to procedural knowledge and Problems 12-16 were part of cluster for assessing principled-conceptual type of knowledge. Statistics of students' results of the experimental and control groups in problems related to each cluster are given in Table 9 and Table 10 respectively.

Table 9. Statistics of results of students of the experimental group regarding four types of knowledge in the knowledge test.

		Intuitive	Concrete Type	Procedural	Principled- conceptual
N	Valid	104	104	104	104
	Missing	0	0	0	0
Mean		1.6202	3.5673	6.3846	3.9038
Median		2.0000	4.0000	6.5000	4.0000
Mode		2.00	4.00	6.00	4.00

Table 10. Statistics of results of students of the control group regarding four types of knowledge in the knowledge test.

		Intuitive	Concrete Type	Procedural	Principled-conceptual
N	Valid	110	110	110	110
	Missing	0	0	0	0
Mean		1.3818	3.3091	5.4636	3.0500
Median		1.0000	3.0000	5.5000	3.0000
Mode		1.00	4.00	5.00	3.00

A Mann-Whitney U test conducted for students' results regarding four types of mathematical knowledge shows that there is a significant difference between students' success of the experimental and control groups in solving problems related to each type of mathematical knowledge that were part of the knowledge test (Table 11).

Table 11. A Mann-Whitney U test for students' results regarding four types of mathematical knowledge in the knowledge test.

	Intuitive	Concrete-Type	Procedural	Principled-conceptual
Mann-Whitney U	4326.000	4628.000	3937.000	3892.500
Wilcoxon W	10431.000	10733.000	10042.000	9997.500
Z	-3.368	-2.718	-3.959	-4.077
Asymp. Sig. (2-tailed)	.001	.007	.000	.000

a. Grouping Variable: Class

Results of students of the experimental group were significantly higher (*Mdn* = 2) than results of students of the control group (*Mdn* = 1) in solving problems related to intuitive knowledge (U = 4326.00, $p < .005$); they were significantly higher (*Mdn* = 4) than results of students of the control group (*Mdn* = 3) in solving problems related to concrete type of knowledge (U = 4628.000, $p < .005$). The same situation was regarding two other types of mathematical knowledge.

Students of the experimental group achieved significantly higher success (*Mdn* = 6.5) than students of the control group (*Mdn* = 5.5) in solving problems related to procedural knowledge (U = 3937.00, $p < .005$) and students of the experimental group outperformed significantly (*Mdn* = 4) their counterparts of the control group (*Mdn* = 3) in solving problems related to principled-conceptual type of knowledge (U = 3892.50, $p < .005$).

The analysis above shows that the implementation of the PGBE model in classes of the experimental group was more effective in promoting students' proficiency in solving problems related to four types of mathematical knowledge about percentages. A comprehensive analysis of students' work in solving problems related to four types of mathematical knowledge is given below.

The intuitive type of knowledge

It is important to note that even in Problems 1 and 2 of the knowledge test (see Appendix) that were related to real-life situations, which could be solved "intuitively" by understanding the language used in them and through simple reasoning, students of the experimental group outperformed those of the control group. Students of the experimental group had an advantage with 20 % more correct answers than those of the control group in solving Problem 2 for calculating the total number of tickets sold, where the whole was a "friendly" number of 100 tickets (Figure 8). The difference in solving a similar problem in the retention test is doubled (40 %) in favor of students of the experimental group (Figure 9).

The concrete type of knowledge

Regarding other problems of both tests that were related to concrete-type of knowledge, such as Problems 3-5 of the knowledge test (see Appendix), students of the experimental group outperformed their counterparts in solving 3 out of 4 items that were concretized with different external representations (Table 12).

Table 12. Students' success in problems related to concrete type of knowledge in the knowledge test and retention test.

Representation of percentages	**Knowledge test [%]**		**Retention test [%]**	
	Exp. group	**Cont. group**	**Exp. group**	**Cont. group**
Etiquette of goods	75.0	72.7	76.9	62.4
Identifying percentage of a shaded part in a geometric figure	96.2	75.5	88.5	81.7
	94.2	87.3	86.5	91.7
Shading part based on a given percentage in a geometric figure	91.3	95.5	93.3	90.8

From Table 12 it can be seen that the small advantage with 2.3 % in favor of students of the experimental group in reading percentages in etiquette of goods in Problem 3 of the knowledge test was increased at 14.5 % in a similar problem of the retention test.

It is important to mention that students of the experimental and control groups used different approaches for solving percentage problems related to shading parts of geometric figures that were part of two items of Problem 4 (see Appendix).

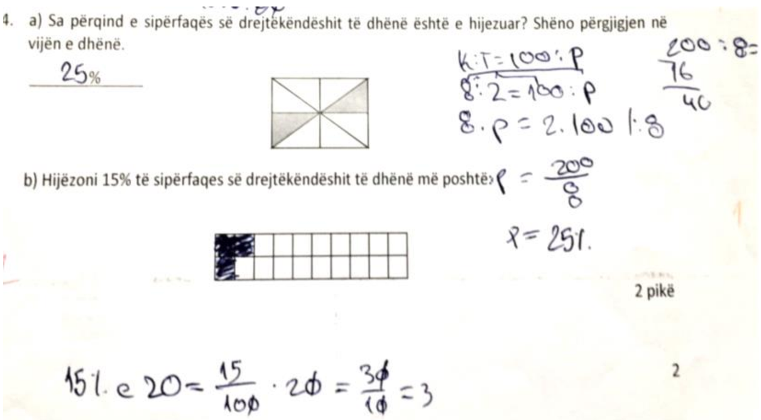


Figure 10. A solution of items 4a and 4b of Problem 4 by a student from the control group through different formulas.

Students of the control group used different formulas, that are characteristic for the traditional way of teaching and learning percentages, for solving items 4a and 4b of Problem 4 (Figure 10), whereas students of the experimental group were more creative, and, additional to different formulas, they also implemented the novel approaches, such as: the percent benchmarks and bar models for solving items 4a and 4b of Problem 4 (Figure 11).

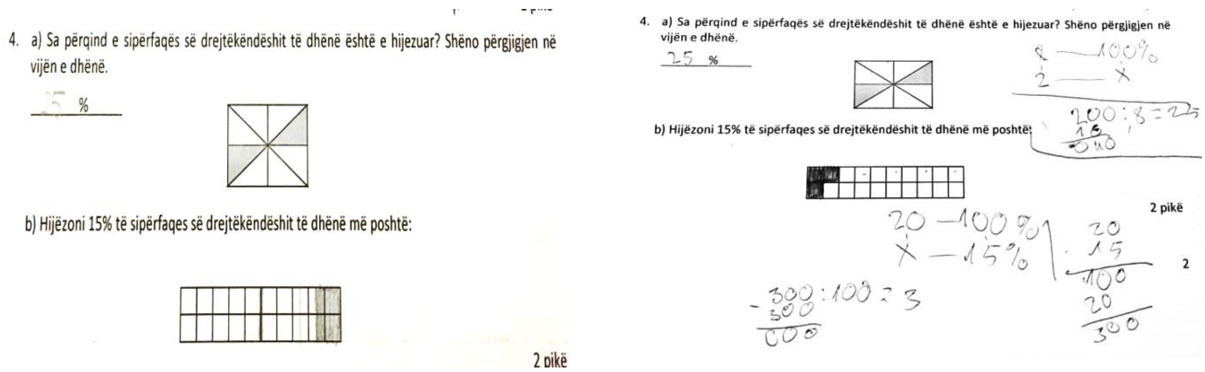


Figure 11. Two solutions of items 4a and 4b of Problem 4 by two students of the experimental group through 10 % - percent benchmarks (left) and bar model (right).

The procedural type of knowledge

As for the procedural type of knowledge, there were two kinds of problems in both tests to assess students' procedural knowledge about percentages. The first kind of problems, such as Problems 6 and 7(see Appendix) were related to mutual conversion of percentages into other multiplicative structures, such as fractions and decimal numbers, whilst the second kind of problems, such as Problems 8-11(see Appendix) were those that required from students to calculate one of the missing referents in the percentage problems. Students of the experimental group outperformed their counterparts in the knowledge test with a difference between 4.6 % - 28 % in solving all items related to mutual conversion of fractions and decimal numbers into percentages (Figure 8). As for solving similar problems of the retention test, students of the control group achieved slightly better success with the maximum advantage of 4.5 % in solving 3 out of 4 items related to mutual conversion of fractions into percentages, whereas students of the experimental group have maintained their remarkably higher advantage with the difference up to 21.6 % in solving 4 items related to mutual conversion of decimal numbers into percentages (Figure 9). A higher result of students of the control group in solving 3 out of 4 items related to mutual conversion of fractions into percentages shows that the traditional way of teaching demonstrates also some benefits (what is also expected) in supporting students' learning, but its implementation influences really a small success compare to the implementation of the PGBE model in classes of the experimental group, where students achieved a higher success with a considerably higher difference in solving other items related to mutual conversion of multiplicative structures in the retention test.

The advantage of students of the experimental group in using percent benchmarks in calculating percentages is reflected in solving both items of Problem 8 (see Appendix). The advantage of students of the experimental group in solving both items of Problem 8 with 15 % difference in the knowledge test (Figure 8) was increased to 18.4 % in the retention test (Figure 9). From Figure 12 it can be seen how much the use of the percent benchmark, within the PGBE model, simplified the work of students of the experimental group in solving both items of Problem 8. On the other hand, students of the control group had to do a lot of calculations for solving the same items of Problem 8 by using the two-step method (Figure 13).

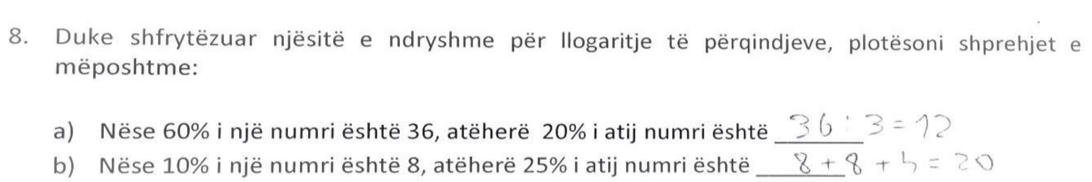


Figure 12. A solution of items 8a and 8b of Problem 8 by a student from the experimental group.

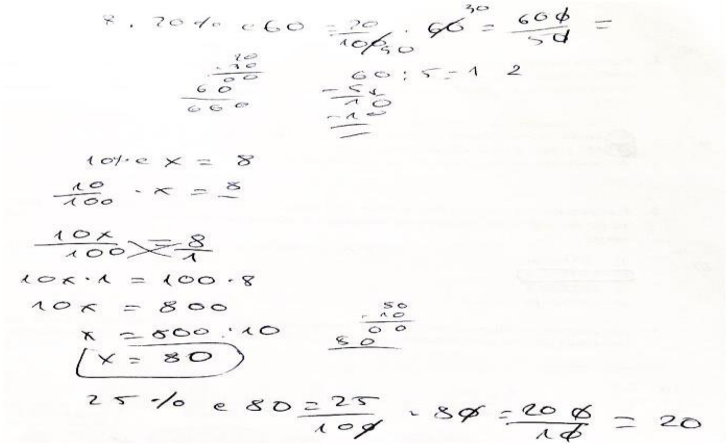


Figure 13. A solution of items 8a and 8b of Problem 8 by a student from the control group.

The introduction of the bar model for building the concept of percentage as a proportional relationship between part and the whole within the PGBE model resulted with higher success of students of experimental group with a difference of 6 % and 10 % in solving problems of Type I and Type II respectively, such as Problems 9 and 10 (see Appendix), and in achieving almost a similar result as students of the control group in solving problem of Type III, such as Problem 11 (Table 13). In the retention test, students of experimental group outperformed their counterparts in solving 2 out of 3 types of problems on percentage, such as Type I and Type III problems (Table 13).

Table 13. Students' results in calculating the referents of percentage problems in the knowledge test and retention test.

Three types of percentage problems	**Knowledge test [%]**		**Retention test [%]**	
	Exp. group	**Cont. group**	**Exp. group**	**Cont. group**
Type I - Calculate part	91.3	85.5	74.0	58.7
Type II - Calculate percent	73.1	63.6	57.7	63.3
Type III - Calculate the whole	48.1	49.1	45.2	44.0

The principled-conceptual knowledge

Regarding the multifaceted meaning of percentages, that is related to principled-conceptual knowledge about them, students of the experimental group were better in the knowledge test than their counterparts with a difference of 15 % in solving two items of Problem 12 (see Appendix) related to the meaning of percentage as a ratio between two parts of the same set, and with 24 % more correct answers in solving the other two items of Problem 12 that were related to the meaning of percentage as a ratio between two different sets (Figure 8). Although these differences are narrowed in a similar problem of the retention test, again students of the experimental group outperformed their counterparts in 3 out of 4 items (Figure 9). Students of the experimental group achieved better success in solving also percentage word problems that were related to various real-life situations, such as in solving Problems 13-15 of the knowledge test (see Appendix). They outperformed students of the control group in solving all percentage word problem with a difference up to 10 % in the knowledge test (Figure 8) and in solving 2 out of 3 of them in the retention test (Figure 9).

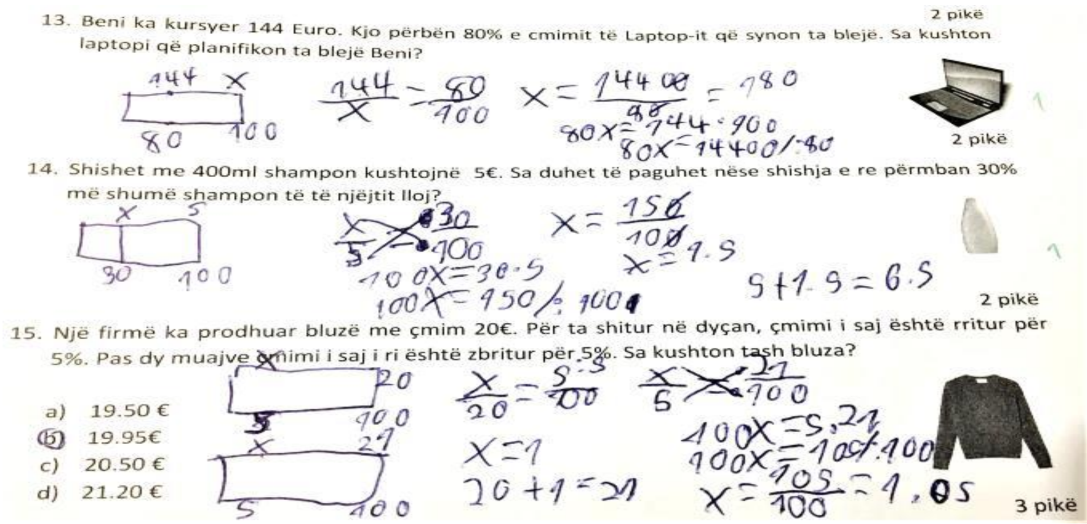
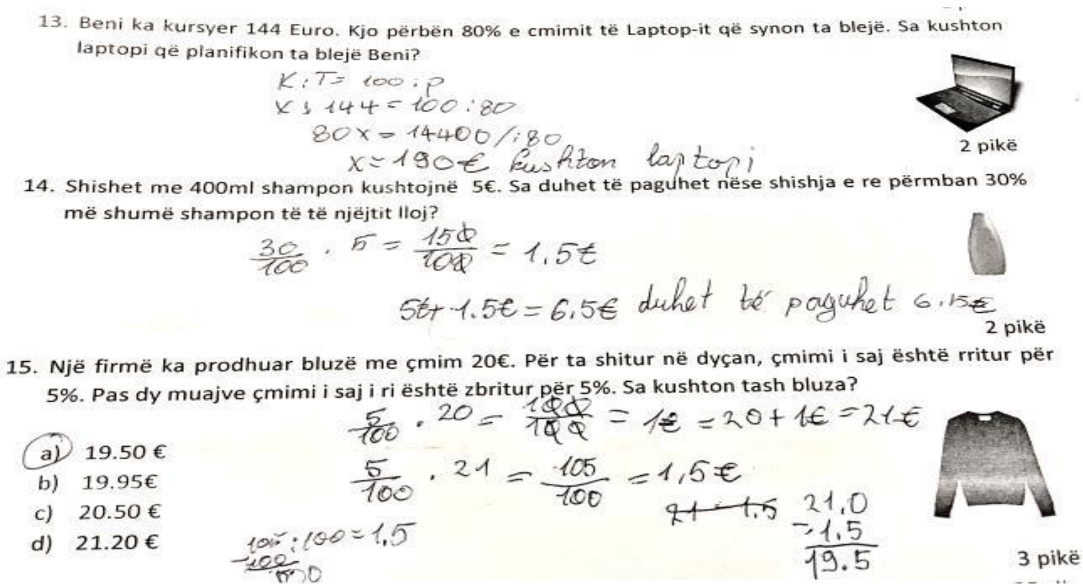


Figure 14. A solution of Problems 13-15 by a student of the experimental group through the bar model.

It is important to mention that the majority of students of the experimental group used bar model in solving all types of percentage word problems (Figure 14), whereas those of the control group used proportional formulas for solving them, which characterizes the traditional approach for solving percentage problems (Figure 15).


13. Beni ka kursyer 144 Euro. Kjo përbën 80% e cmimit të Laptop-it që synon ta blejë. Sa kushton laptopi që planifikon ta blejë Beni?

K:T = 100:p
x : 144 = 100 : 80
80x = 14400 /:80
x = 180€ kushton laptopi

2 pikë

14. Shishet me 400ml shampon kushtojnë 5€. Sa duhet të paguhet nëse shishja e re përmban 30% më shumë shampon të të njëjtit lloj?

$\frac{30}{100} \cdot 5 = \frac{150}{100} = 1.5€$

5€ + 1.5€ = 6.5€ duhet të paguhet 6.15€

2 pikë

15. Një firmë ka prodhuar bluzë me çmim 20€. Për ta shitur në dyçan, çmimi i saj është rritur për 5%. Pas dy muajve çmimi i saj i ri është zbritur për 5%. Sa kushton tash bluza?

a) 19.50 €
b) 19.95€
c) 20.50 €
d) 21.20 €

$\frac{5}{100} \cdot 20 = \frac{100}{100} = 1€ = 20 + 1€ = 21€$

$\frac{5}{100} \cdot 21 = \frac{105}{100} = 1,5€$

105 : 100 = 1.5

21.0 − 1.5 = 19.5

3 pikë

Figure 15. A solution of Problems 13-15 by a student of the control group through the use of different formulas.

Since creating a problem and solving it is considered as one of the strategies for assessing students' comprehension of a certain concept in mathematics (Alibali, 1999; Ambrose et al., 2010), students were asked to write a problem on percentage based on given numbers and solve it (Problem 16). From Table 14, it can be seen that students of the experimental group outperformed their counterparts in formulating a problem based on given percentage and in solving it in both tests.

Table 14. Students' results in formulating a problem based on given percentage and in solving it.

	Knowledge test [%]		Retention test [%]	
	Exp. group	**Cont. group**	**Exp. group**	**Cont. group**
Formulate the problem based on given percentage	79.8	65.5	64.4	56.0
Solve the formulated problem	49.0	30.9	42.3	33.0

In general, from students' results in both tests, it is evident that students of experimental group outperformed their counterparts in 92.9 % of all items of the knowledge test and in 75 % of all items of the retention test conducted two months later. It is interesting to emphasize that the maximal difference of students of the experimental group compared to those of control group in both tests was 40 %, whereas those of control group outperformed their counterparts maximally with 7 % difference. This shows that the implementation of the PGBE for teaching and learning percentages had impacted the learning of students of Grade 7, since it influenced their in-depth learning and a long-lasting knowledge about percentages.

Conclusion

The goal of this research study was to design a new learning model for supporting students' in-depth understanding of percentages and assess the effectiveness of its implementation in Kosovo context by organizing two cycles of piloting and conducting the experimental method in ten different schools in Kosovo. Piloting offered the possibility to assess the effectiveness of the interrelation of different research-based instructional models into the development path of the HLT 1 and HLT 2 and the social interaction between teacher and students, and also between students themselves during their implementation. Through piloting it was possible to get a better landscape about the implementation of the HLTs in different learning communities. The feedback from consecutive piloting of the HLT 1 and HLT 2 was used to finalize the PGBE model as a well-functioning model for building students' mathematical knowledge about percentages, mastering their computational skills, and sophisticating their mathematical thinking and reasoning about percentages.

Even though the piloting was done twice, it was considered important to assess the effectiveness of the implementation of the PGBE model by conducting the experimental method with students of Grade 7. For this reason, the knowledge test and retention test on percentages was organized for 214 students of eight Grade 7 classes of eight different schools from two municipalities. Students' results in both tests show that students of the experimental group outperformed students of the control group in 26 out of 28 items of the knowledge test and in 21 out of 28 of items of the retention test conducted two months later, at the end of the school year. This is a strong indicator proving that the implementation of the PGBE model for building students' understanding of percentages has opened up the way for a more sustainable knowledge of percentages among more students of the experimental group.

Additional to students' results, there is also a positive perception about the implementation of the PGBE model from two teachers who piloted the model in their classes and four other teachers who implemented the model in classes of the experimental group. During semi-structured interviews, teachers mentioned that students used the bar model for calculating percentages not only in mathematics, but also in other school subjects, such as in physics, chemistry, biology and in a school project regarding nutrition, which aimed to raise students' awareness about consuming healthy products through analyzing the percentages of ingredients of foodstuffs.

It was not the intention of authors to design a model that fits to all learning context, but to design an effective learning model for building students' mathematical knowledge about percentages for students of Grade 7 (13 and 14-years-old students), when they enter to the formal operation thought and their cognitive development ensures the conceptual understanding of percentages as proportional statements.

The PGBE model provides new insights for deepening students' understanding of percentages that can be implemented with students of Grade 7 in different learning settings. The PGBE model contributes to the theory of gaining mathematical knowledge, since it presents a new framework for supporting the development progress of students in learning with understanding, mathematical thinking and reasoning of percentages and other multiplicative structures, such as fractions and decimal numbers.

Acknowledgement

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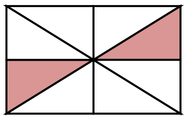
Appendix: Knowledge test on percentages

1. The weather forecast for tomorrow shows that 60 % of day is likely to be rainy, while the rest will be sunny. Which of the following answers are correct for tomorrow? (Round the correct answer. Beware that there is a possibility of having more than one correct answer.)
   a) More than half of the day will be sunny.
   b) During the day there will be more rainy weather than sunny.
   c) Less than half of the day will be sunny.
   d) Rainy weather will be less than sunny.

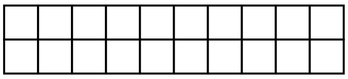
2. Two hours before the start of the film there were sold 100 tickets. The last two hours there were sold 50 % more than two hours before. The total number of ticket sold was (Round the correct answer.)
   a) 50 tickets.
   b) 150 tickets.
   c) 250 tickets.
   d) 350 tickets.

3. The percentage of polyester in the sweater ticket is deleted. Fill in the line below the percentage of polyester in the sweater?
   _____%____

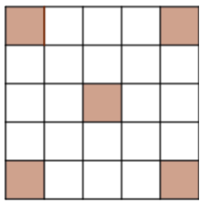
4. a) What percentage of rectangle is shaded? Write the answer on the line provided.
   ______%_____



   b) Shade 15 % of the rectangle given below:



5. Some of the squares in the given figure are shaded. Which of the following answers is correct? Explain your answer.



   a) It is shaded exactly $\frac{5}{20}$ of the figure
   b) It is shaded exactly $\frac{1}{3}$ of the figure
   c) It is shaded approximately 0.25 of the figure
   d) It is shaded approximately 20 % of the figure.

6. Convert given fractions in a) and b) in percentages and given percentages in c) and d) in fractions:
   a) $\frac{3}{10}$ = ______ c) 0.5 % = ______
   b) $\frac{4}{5}$ = ______ d) 113 % = ______

7. Convert the following decimal numbers in a) and b) in percentages and given percentages in c) and d) in decimal numbers:
   a) 0.4 = ______ c) 8 % = ______
   b) 1.25 = ______ d) 1.3 % = ______

8. By applying different benchmarks, calculate percentages below:
   a) If 60 % of a number is 36, then 20 % of the same number is ______
   b) If 10 % of a number is 8, then 25 % of the same number is ______

9. What is 18 % of 750?

10. What percentage of number 140 is number 28?

11. What is the number, if 32 % of it equals to 480?

12. Ratio among girls and boys in a class is 2:3. Link the part of sentences in the left with correct part of it in the right (use arrows to link parts of sentences).

1) The percentage of girls in the class is	a) 150 %
2) The percentage of boys in the class is	b) 66,6 %
3) The percentage of girls compare to boys in the class is	c) 60 %
4) The percentage of boys compare to girls in the class is	d) 40 %

13. Beni saved 144 €. This amount is 80 % of the price of the Laptop that he intends to buy. What is the price of Laptop that Ben intends to buy?

14. Bottles with 400 ml shampoo costs 5 €. How much should you pay if new bottles contain 30 % more of the same shampoo?

15. A firm produced a blouse for 20 €. For selling it in the market, its price was increased 5 %. After two months its new price was decreased 5 %. How much the blouse costs now?
    a) 19.50 €
    b) 19.95 €
    c) 20.00 €
    d) 20.50 €

16. Write a task related to real life situations by using numbers 35 and 120 and the symbol of %. Solve the task you have written.